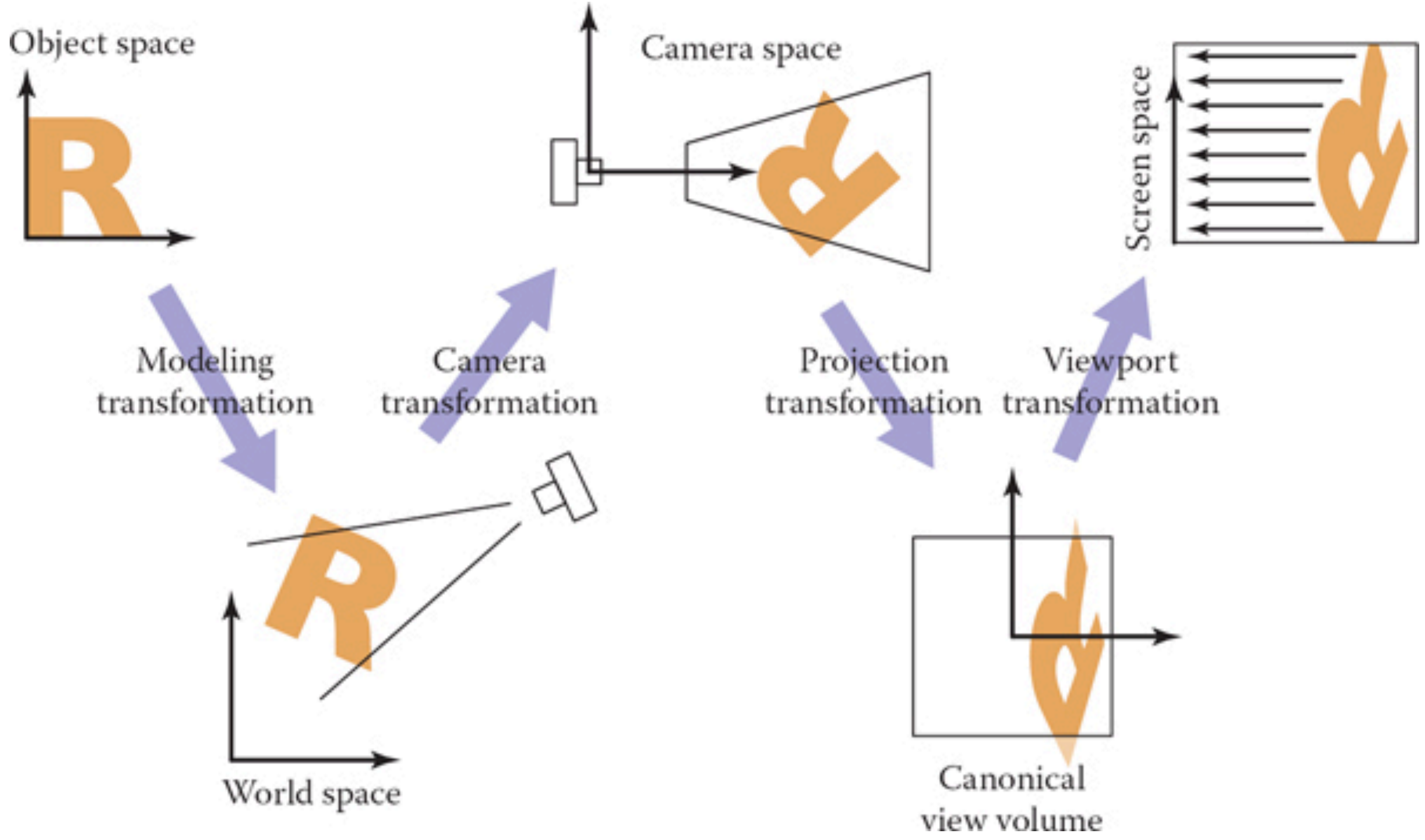
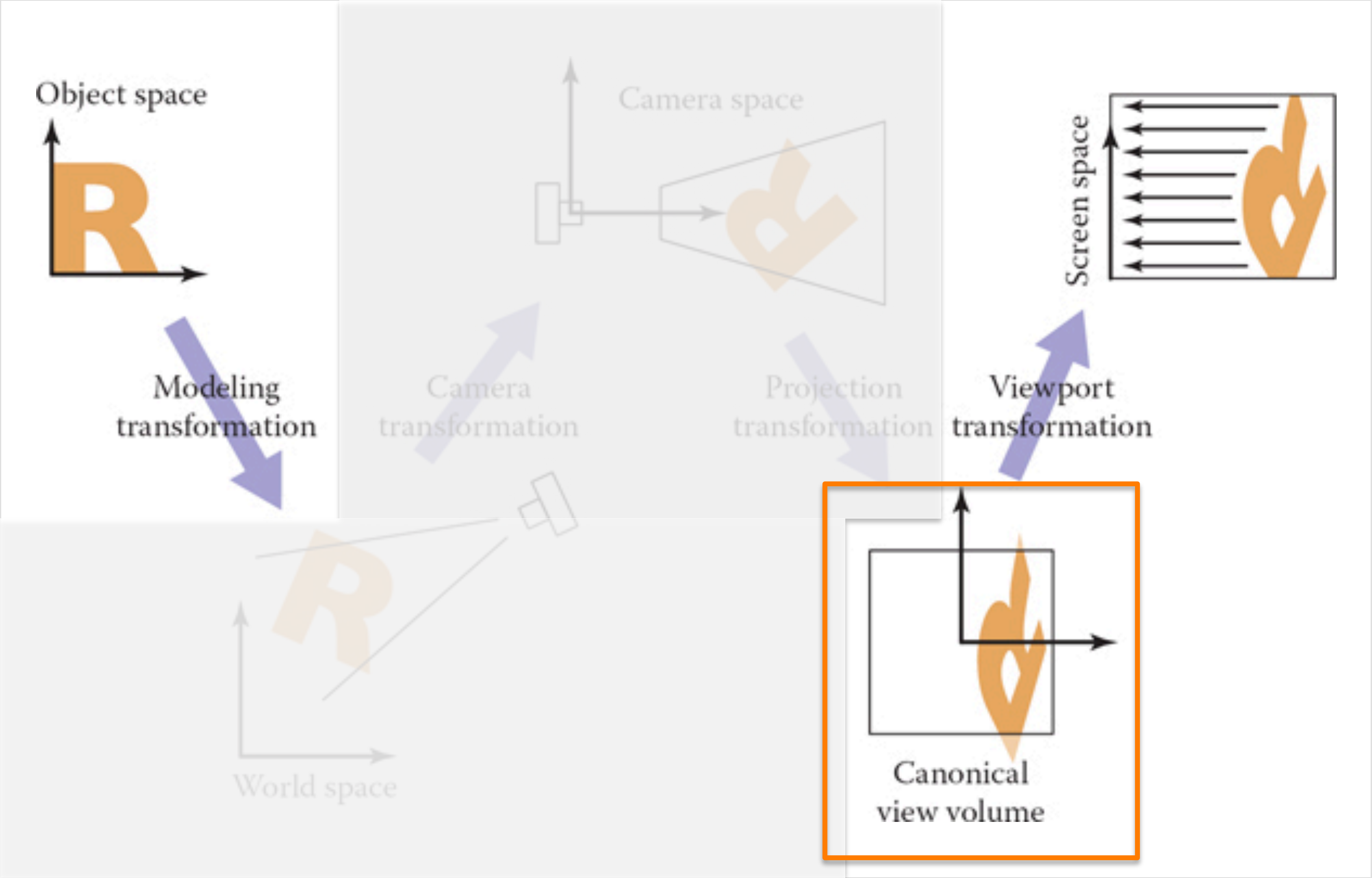
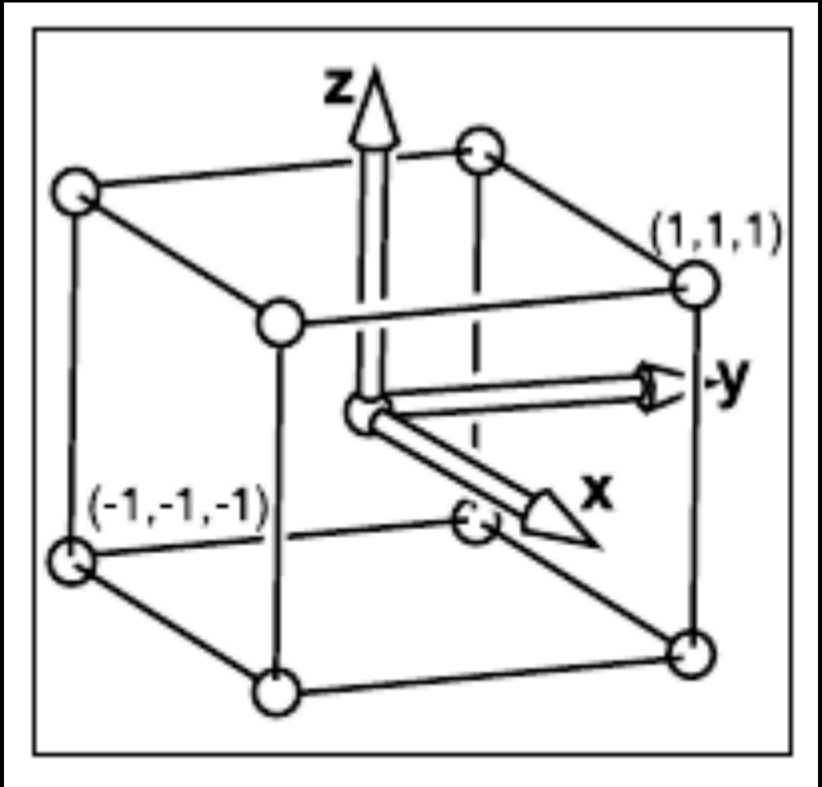
Viewing

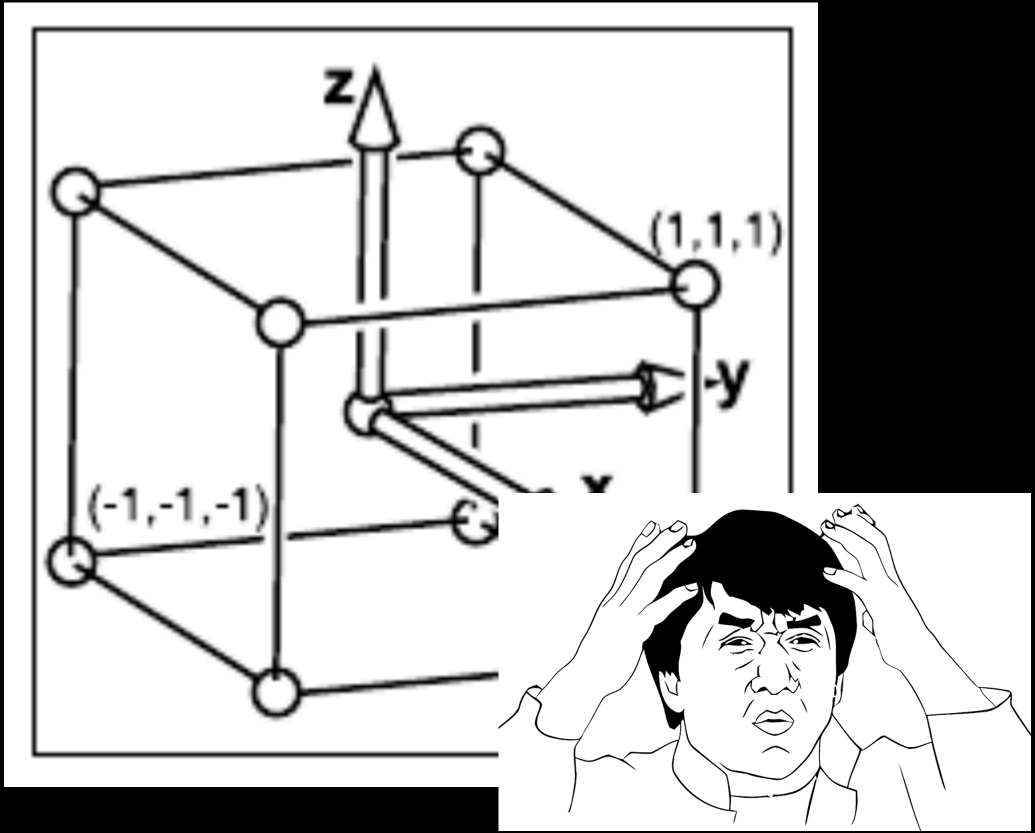
CPSC 453 - Fundamentals of Computer Graphics


Object space
Camera space
Screen space
Modeling transformation
Camera transformation
Projection transformation
Viewport transformation
World space
Canonical view volume


Object space
Camera space
Screen space
Modeling transformation
Camera transformation
Projection transformation
Viewport transformation
World space
Canonical view volume



Normalized Device Coordinates (NDC) : $[-1,1]^3$

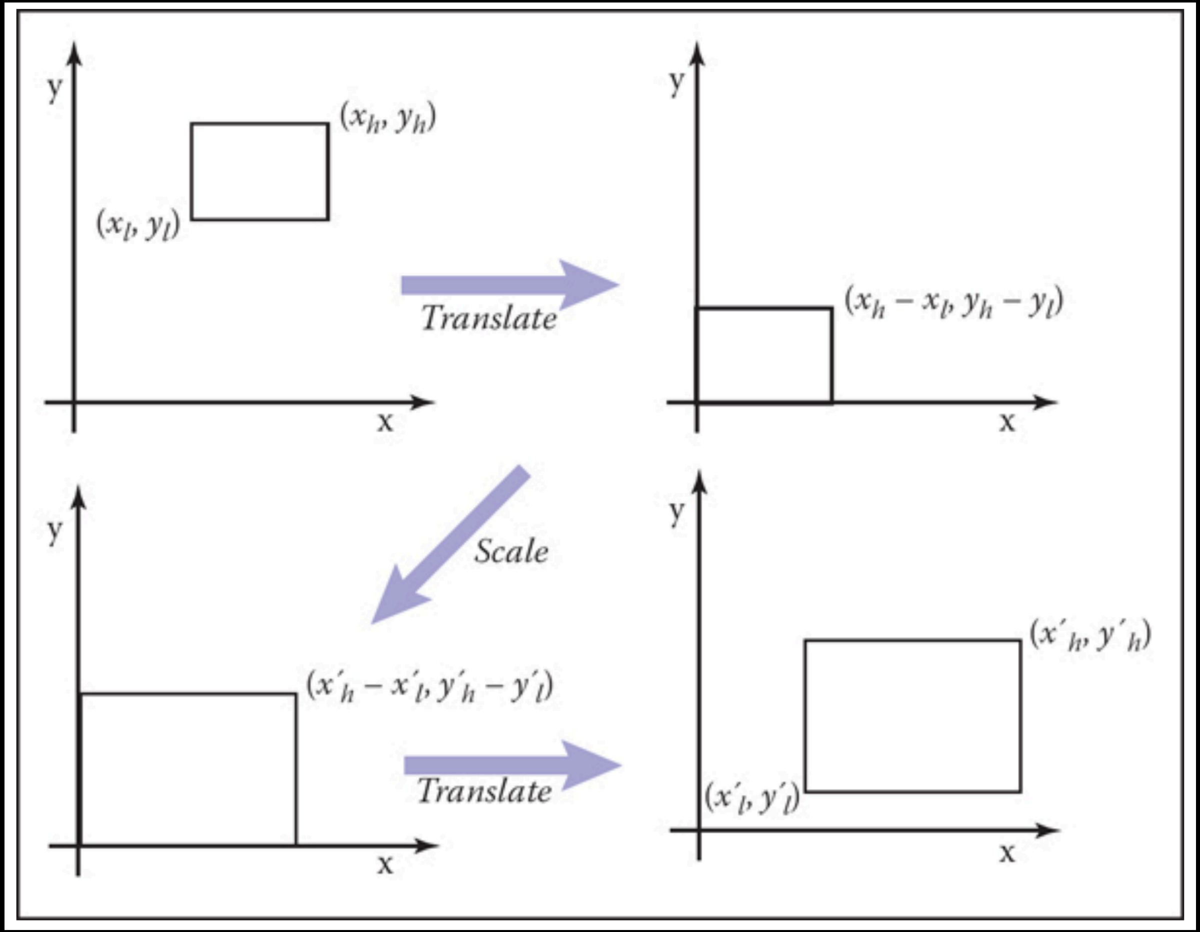


Normalized Device Coordinates (NDC) : $[-1,1]^3$

Scaling in 1D

Mapping in 1D

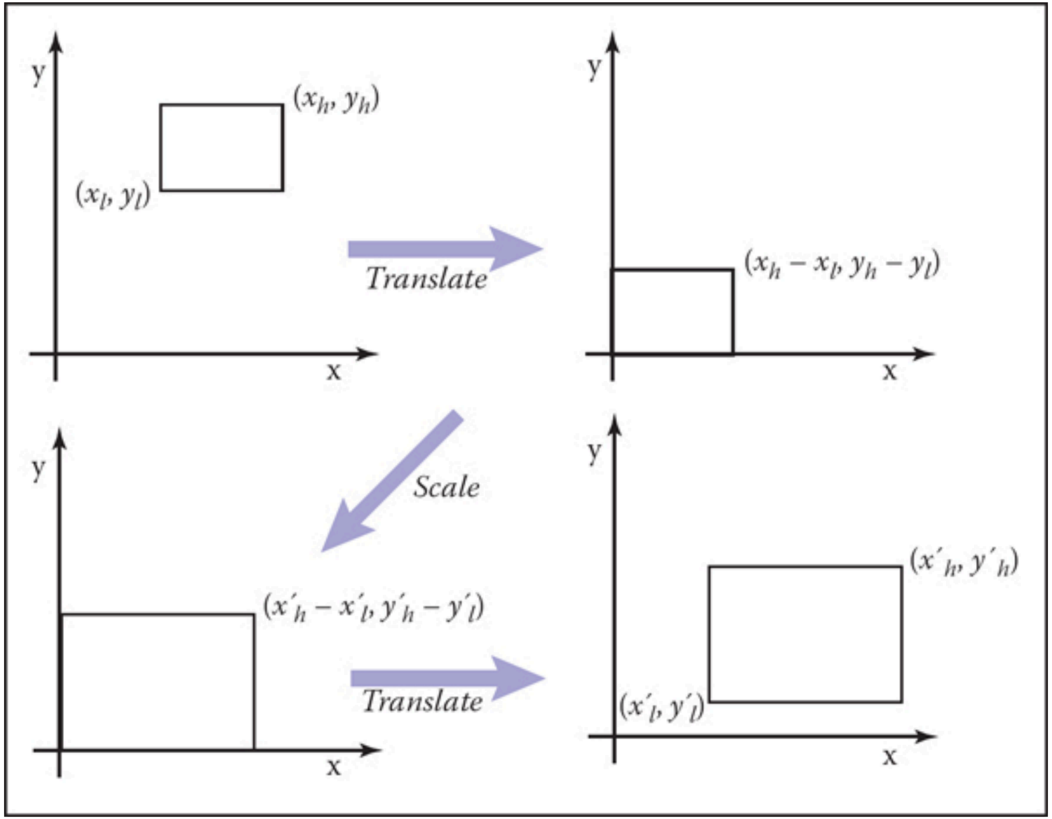
Mapping in 2D



Mapping in 2D

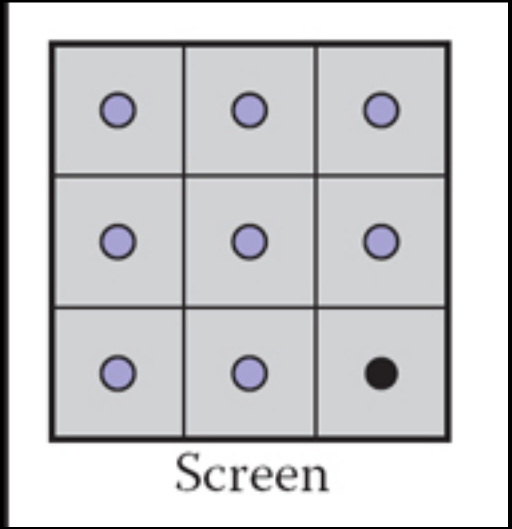
$$\begin{bmatrix}1&0&x'_l\\0&1&y'_l\\0&0&1\end{bmatrix}\begin{bmatrix}\frac{x'_h-x'_l}{x_h-x_l}&0&0\\0&\frac{y'_h-y'_l}{y_h-y_l}&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-x_l\\0&1&-y_l\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}\frac{x'_h-x'_l}{x_h-x_l}&0&\frac{x'_l x_h - x'_h x_l}{x_h-x_l}\\ 0&\frac{y'_h-y'_l}{y_h-y_l}&\frac{y'_l y_h - y'_h y_l}{y_h-y_l}\\0&0&1\end{bmatrix}.$$



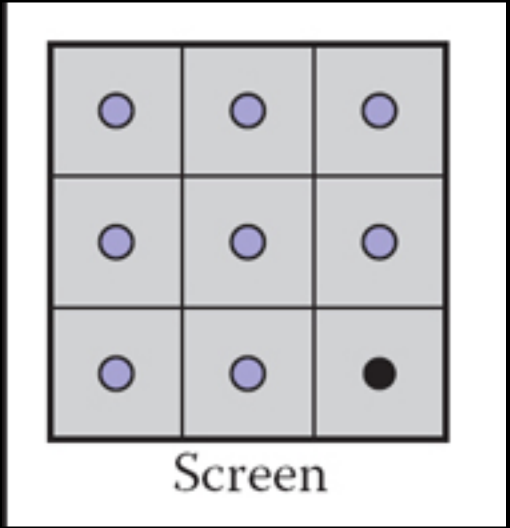
2D mapping NDC to screen space

$$[-1,1]^2 \longrightarrow [-0.5, -n_x - 0.5] \times [-0.5, -n_y - 0.5]$$

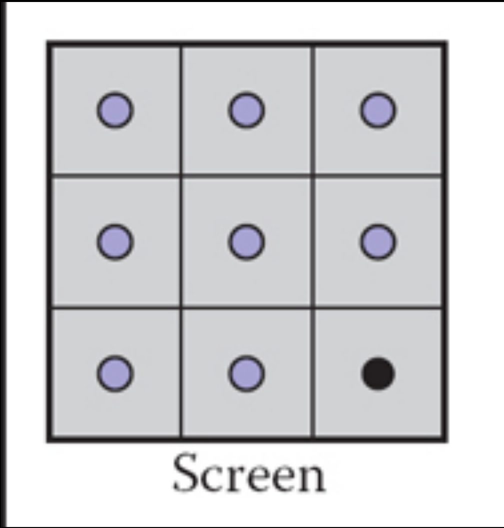


2D mapping NDC to screen space

$$\begin{bmatrix} x_{\text{screen}} \\ y_{\text{screen}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & \frac{n_y - 1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{canonical}} \\ y_{\text{canonical}} \\ 1 \end{bmatrix}$$

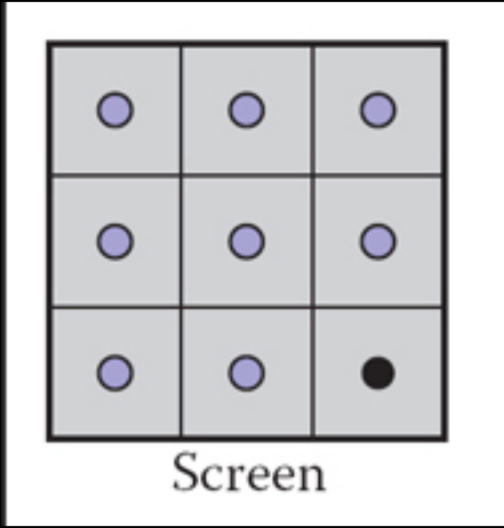


3D mapping: Viewport transformation



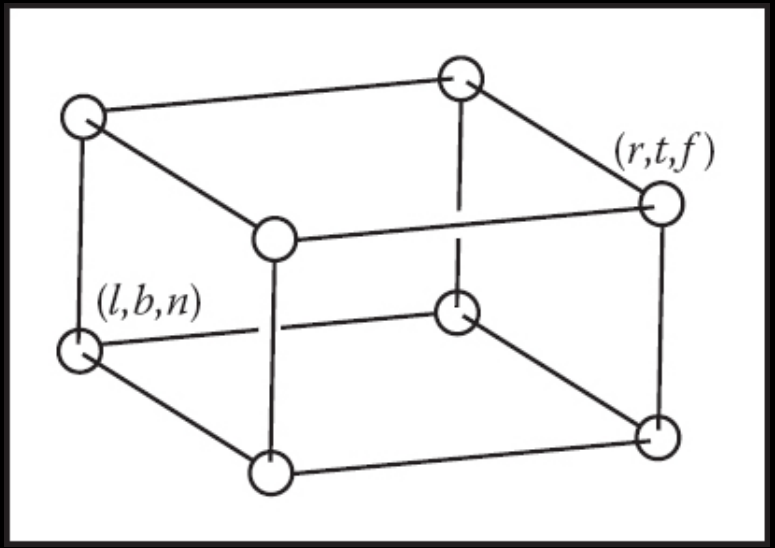
$$M_{\text{vp}} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D mapping: Viewport transformation

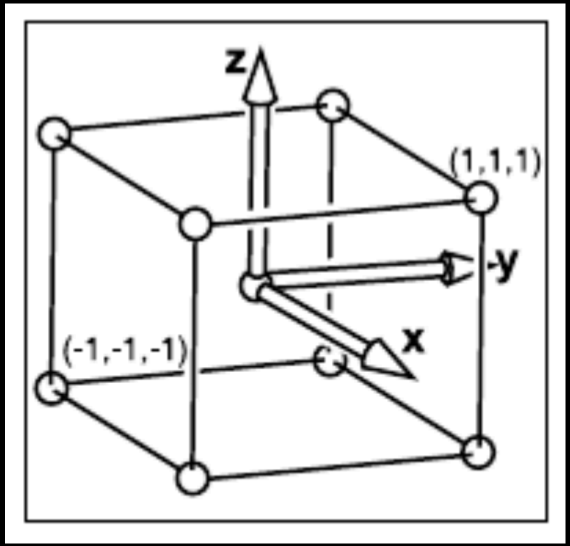


$$M_{\text{vp}} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

OpenGL does this automatically with the values set in
glViewport(x, y, width, height)

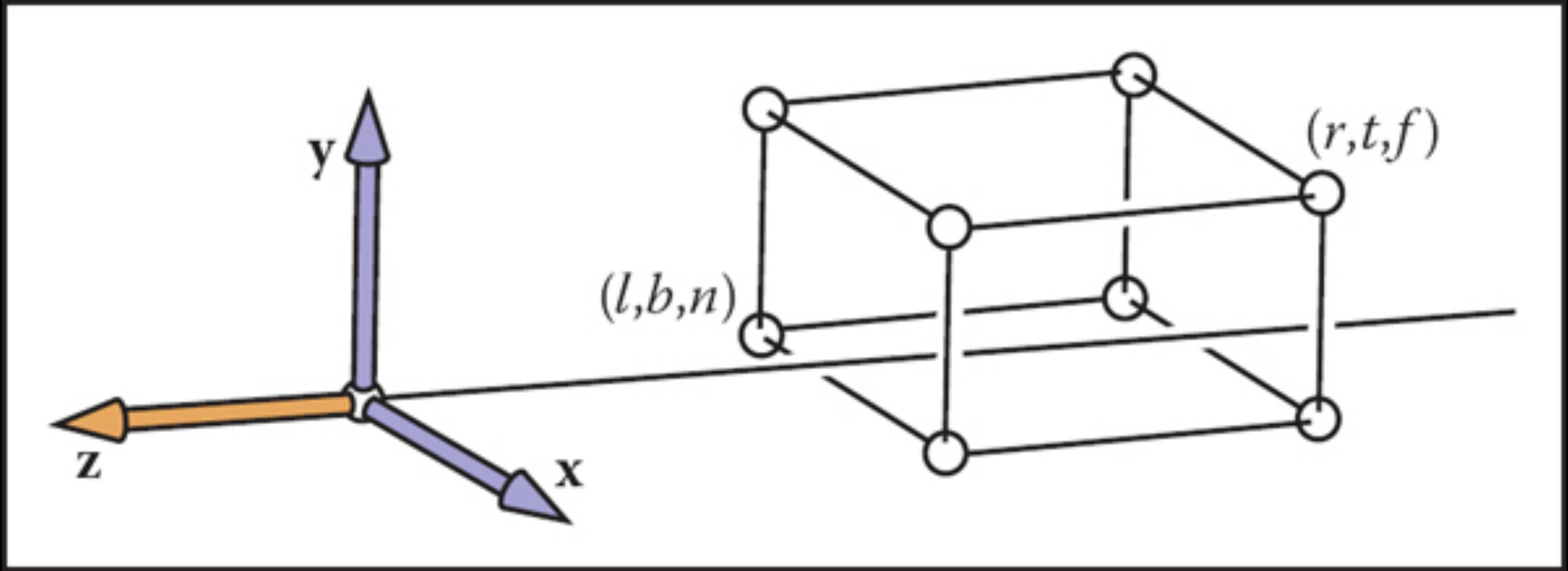


Orthographic view volume



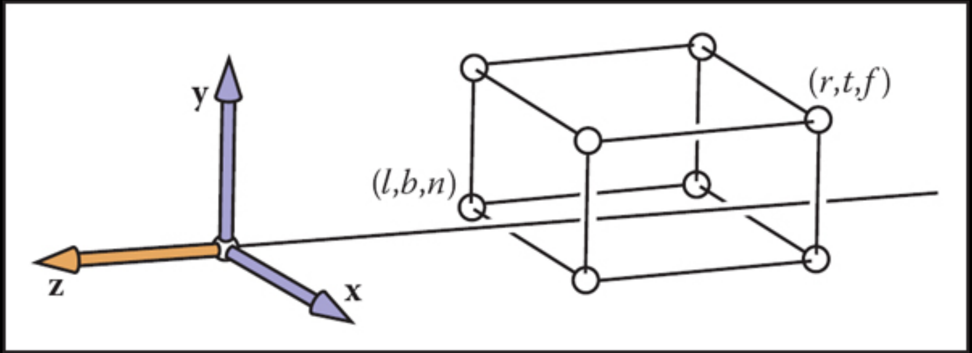
NDC volume

Orthographic view volume



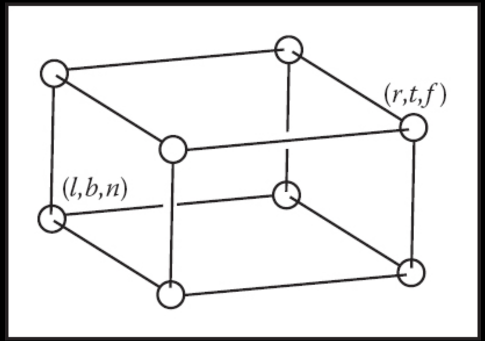
Volume is along z-axis (i.e. $n > f$)

Orthographic view volume

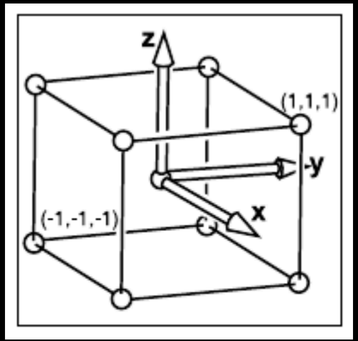


$$
\begin{aligned}
x = l &\equiv \text{left plane},\\
x = r &\equiv \text{right plane},\\
y = b &\equiv \text{bottom plane},\\
y = t &\equiv \text{top plane},\\
z = n &\equiv \text{near plane},\\
z = f &\equiv \text{far plane}.
\end{aligned}
$$

Mapping one box to another…

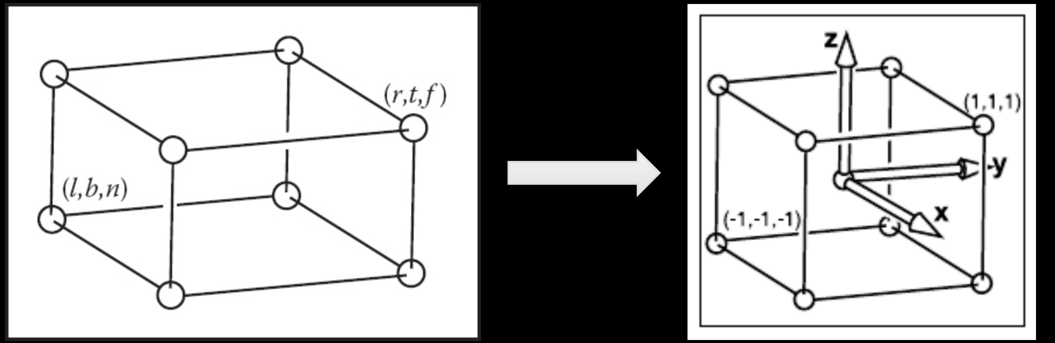


Orthographic view volume



NDC volume

Mapping one box to another…

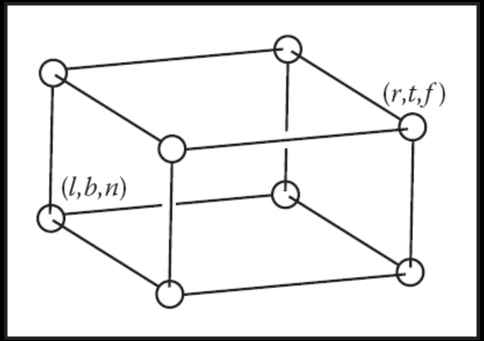


Orthographic view volume

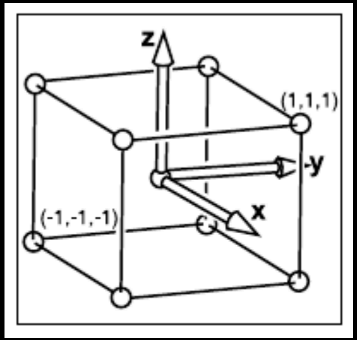
NDC volume

$$\mathbf{M}_{\text{orth}} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

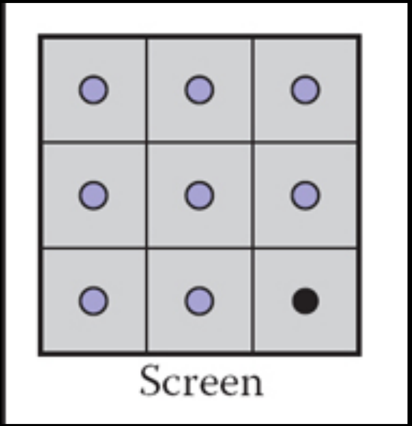
$$\begin{bmatrix} x_{\text{pixel}} \\ y_{\text{pixel}} \\ z_{\text{canonical}} \\ 1 \end{bmatrix} = (\mathbf{M}_{\text{vp}} \mathbf{M}_{\text{orth}}) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



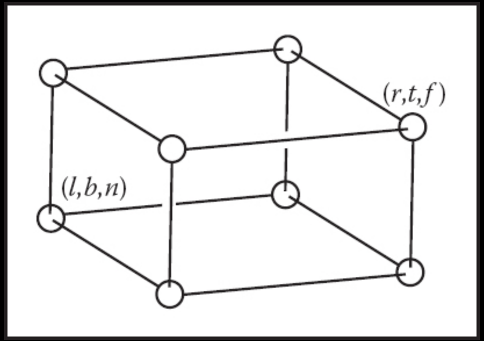
Orthographic view volume



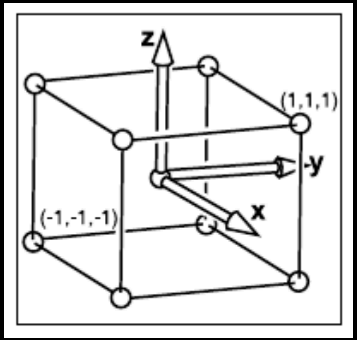
NDC volume



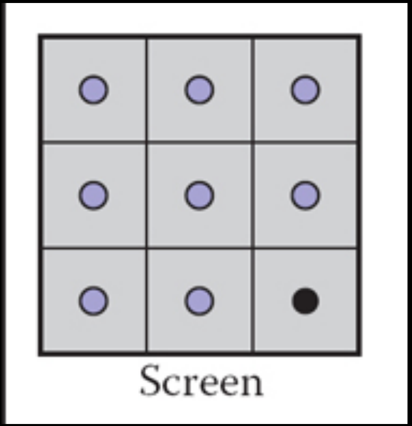
$$\begin{bmatrix} x_{\text{pixel}} \\ y_{\text{pixel}} \\ z_{\text{canonical}} \\ 1 \end{bmatrix} = (\mathbf{M}_{\text{vp}}\mathbf{M}_{\text{orth}}) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Orthographic view volume

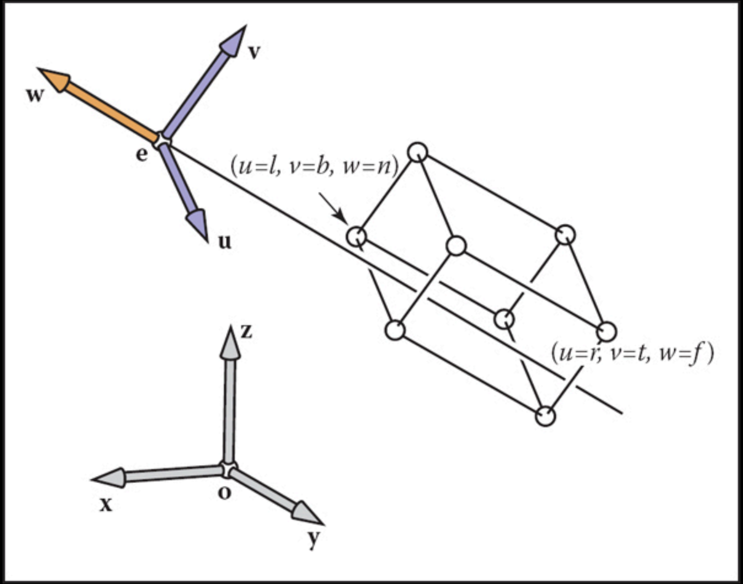


NDC volume

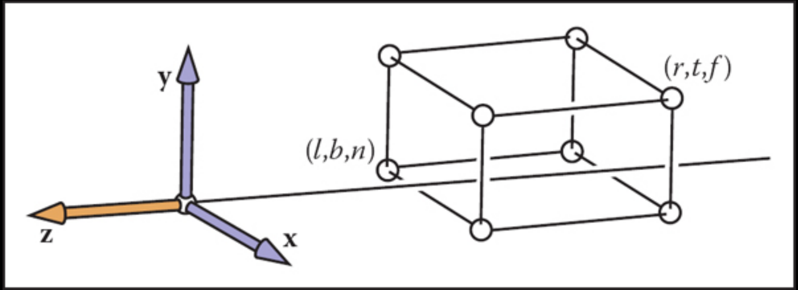


The z-coordinate is in [-1,1], and will be used for depth buffering later.

Camera transformation

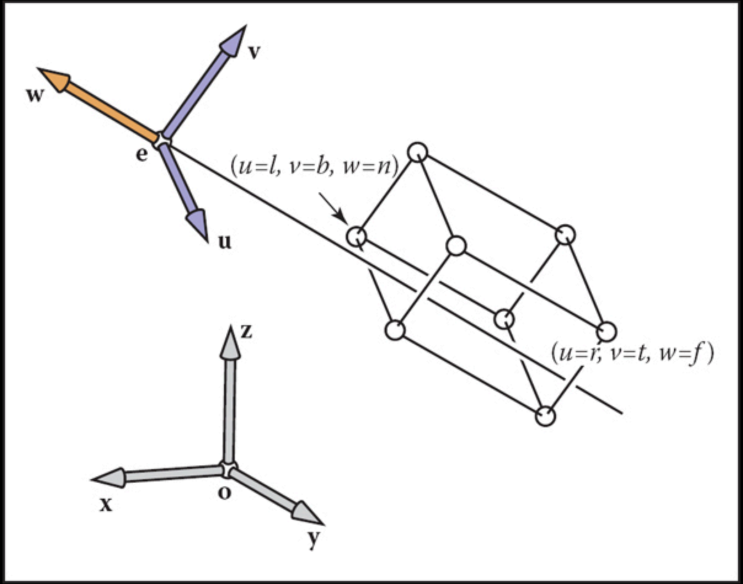


Arbitrary view transform

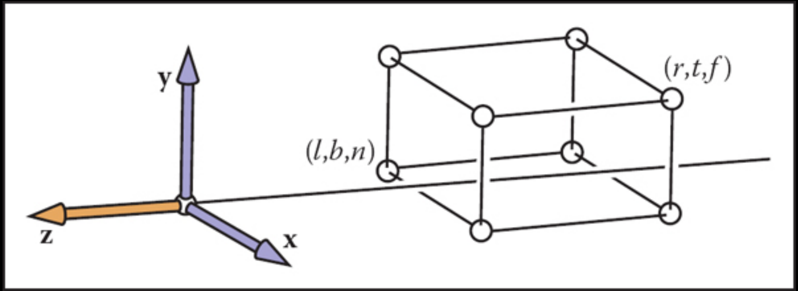


Orthographic view volume

Camera transformation



Arbitrary view transform



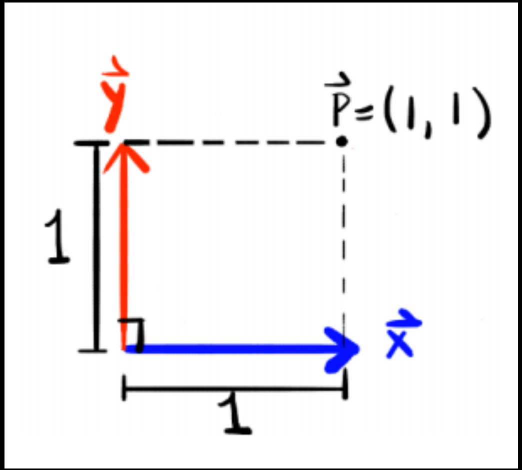
Orthographic view volume

2D change of coordinates

$$\boldsymbol{u} = u_1\boldsymbol{b}_1 + u_2\boldsymbol{b}_2 + \cdots + u_n\boldsymbol{b}_n$$

In $\mathbb{R}^2$, the *standard basis* is $(\boldsymbol{e}_1, \boldsymbol{e}_2)$,

$$(\boldsymbol{e}_1, \boldsymbol{e}_2) = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)$$

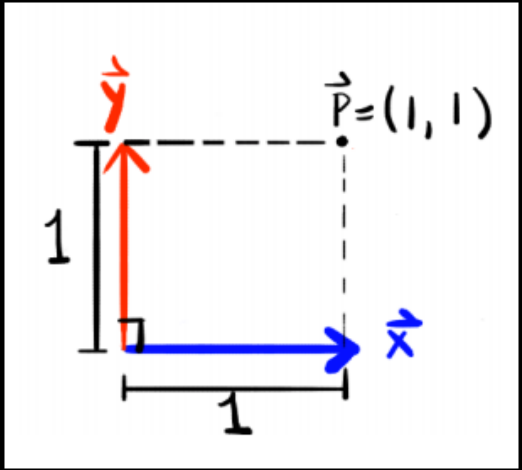


2D change of coordinates

$$\boldsymbol{u} = u_1\boldsymbol{b}_1 + u_2\boldsymbol{b}_2 + \cdots + u_n\boldsymbol{b}_n$$

In $\mathbb{R}^2$, the *standard basis* is $(\boldsymbol{e}_1, \boldsymbol{e}_2)$,

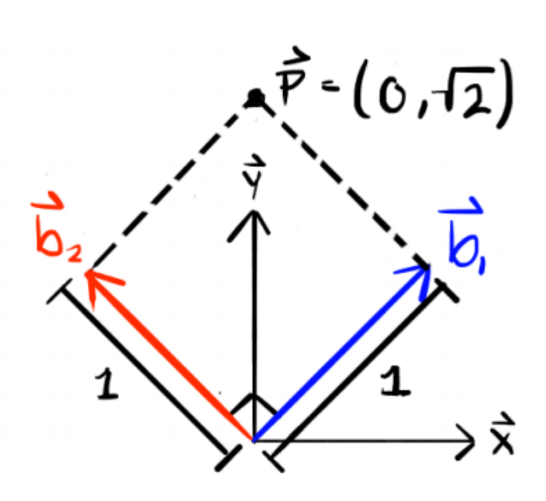
$$(\boldsymbol{e}_1, \boldsymbol{e}_2) = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$



$$Ip = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2D change of coordinates

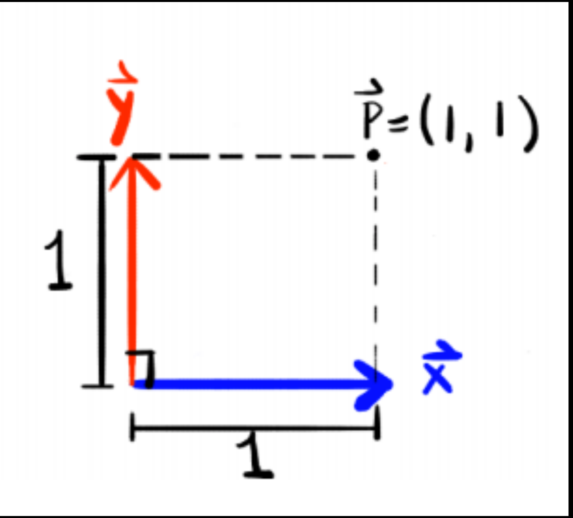
$$(\boldsymbol{b}_1, \boldsymbol{b}_2) = \left(\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \right)$$

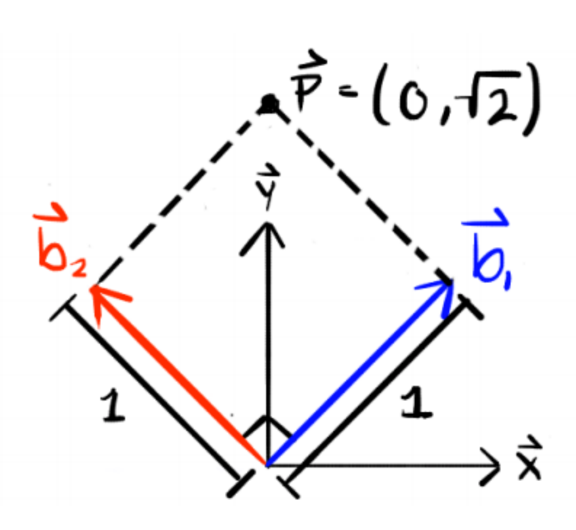


$$B\boldsymbol{p} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

2D change of coordinates

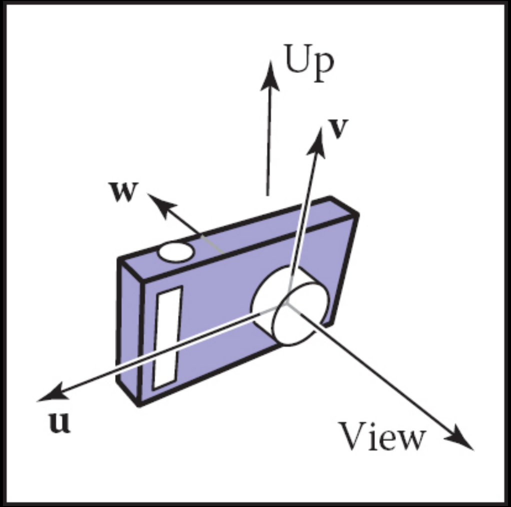
$$B\boldsymbol{p} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

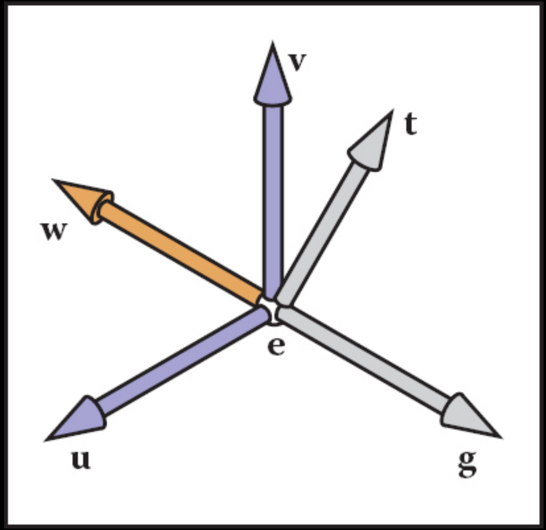




A matrix with *columns* that are mutually *orthonormal*, is a rotation matrix

Camera transformation



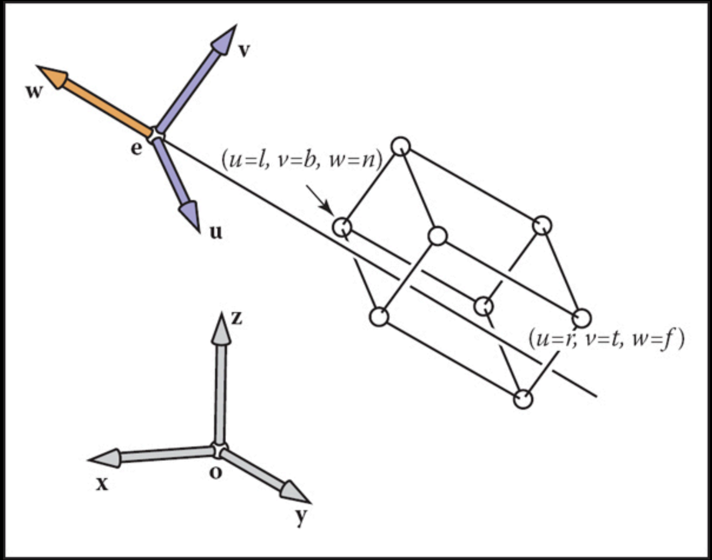


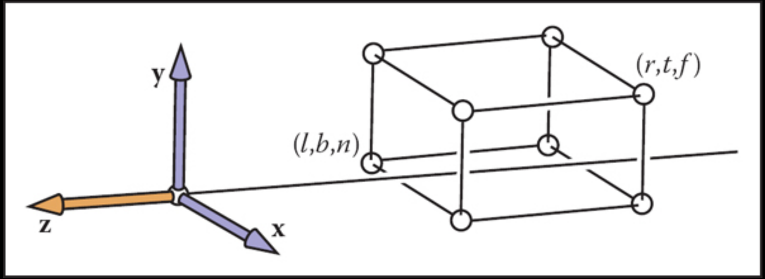
$$\mathbf{w} = -\frac{\mathbf{g}}{\|\mathbf{g}\|},$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|},$$

$$\mathbf{v} = \mathbf{w} \times \mathbf{u}.$$

Camera transformation





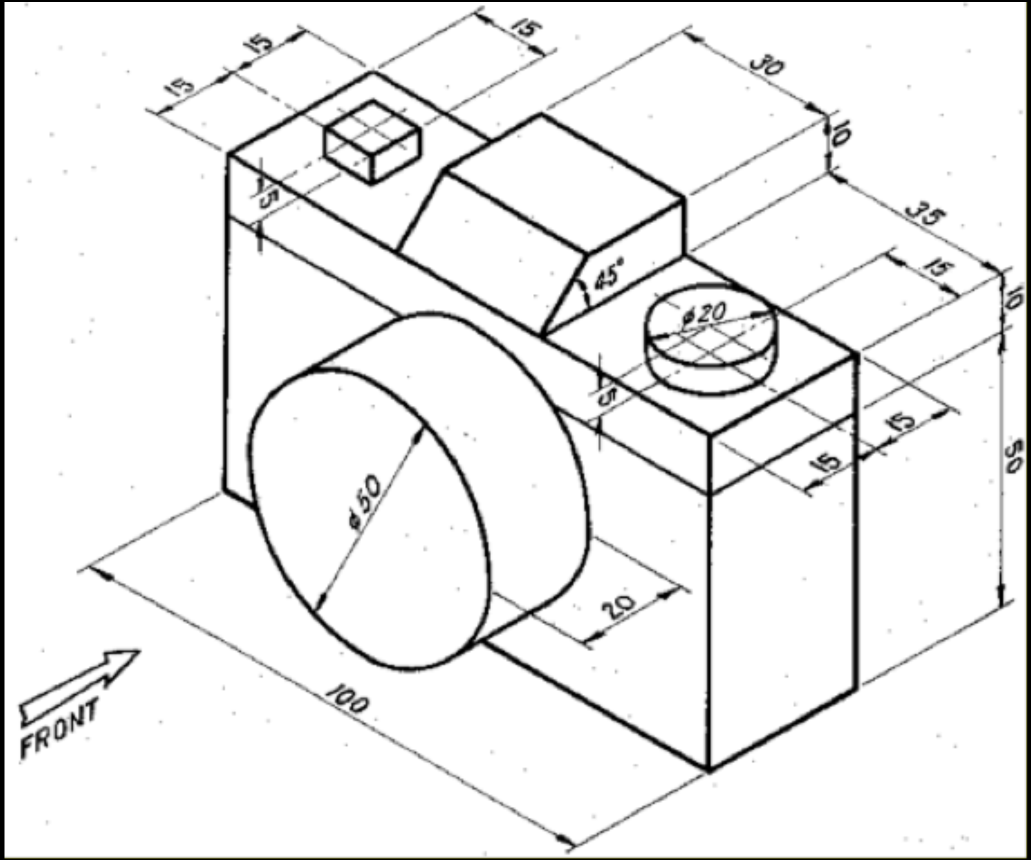
Orthographic view volume

$$\mathbf{M}_{\text{cam}} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

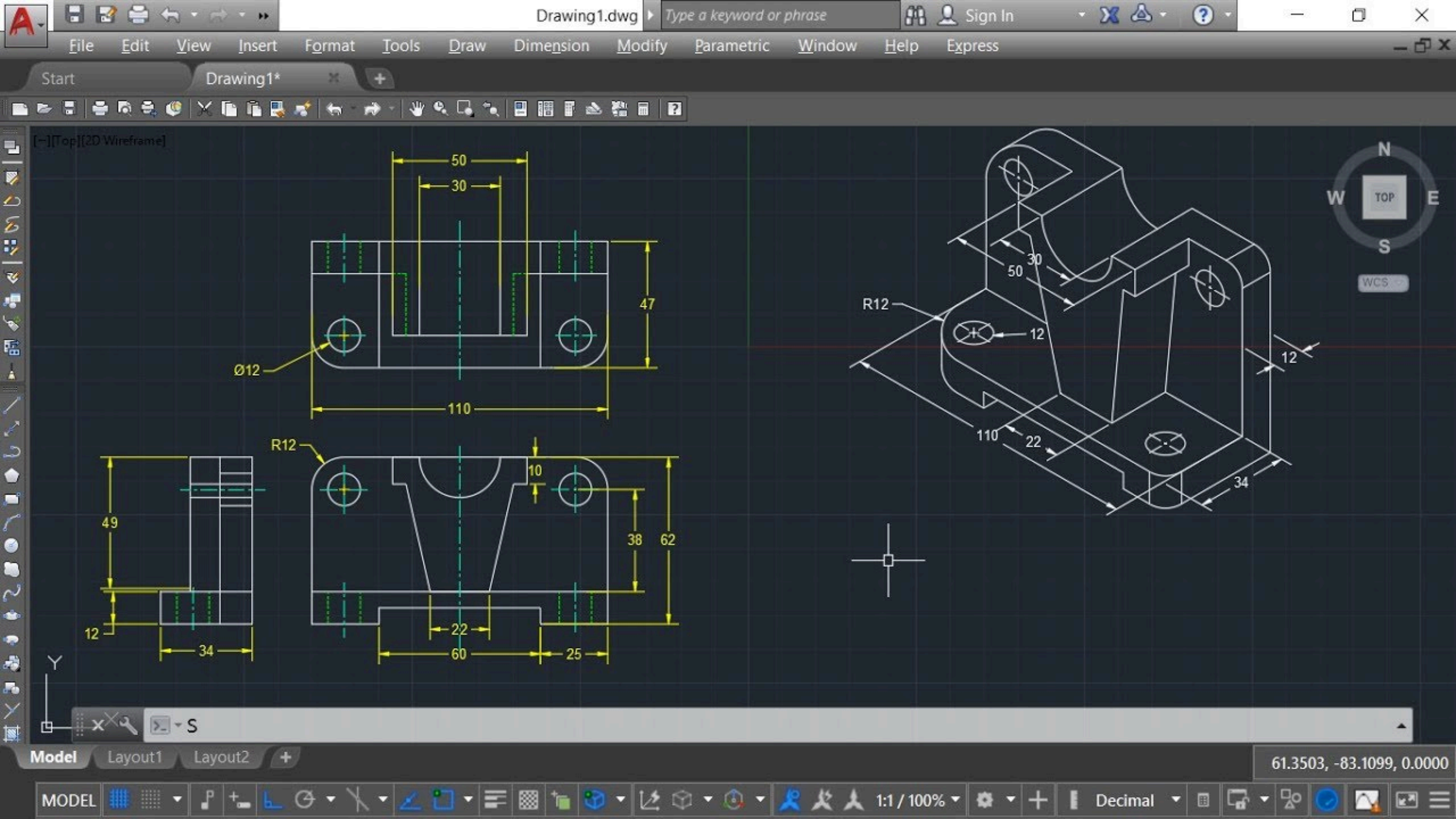
World space to Screen space

$$\mathbf{M} = \mathbf{M}_{\mathrm{vp}} \mathbf{M}_{\mathrm{orth}} \mathbf{M}_{\mathrm{cam}}$$

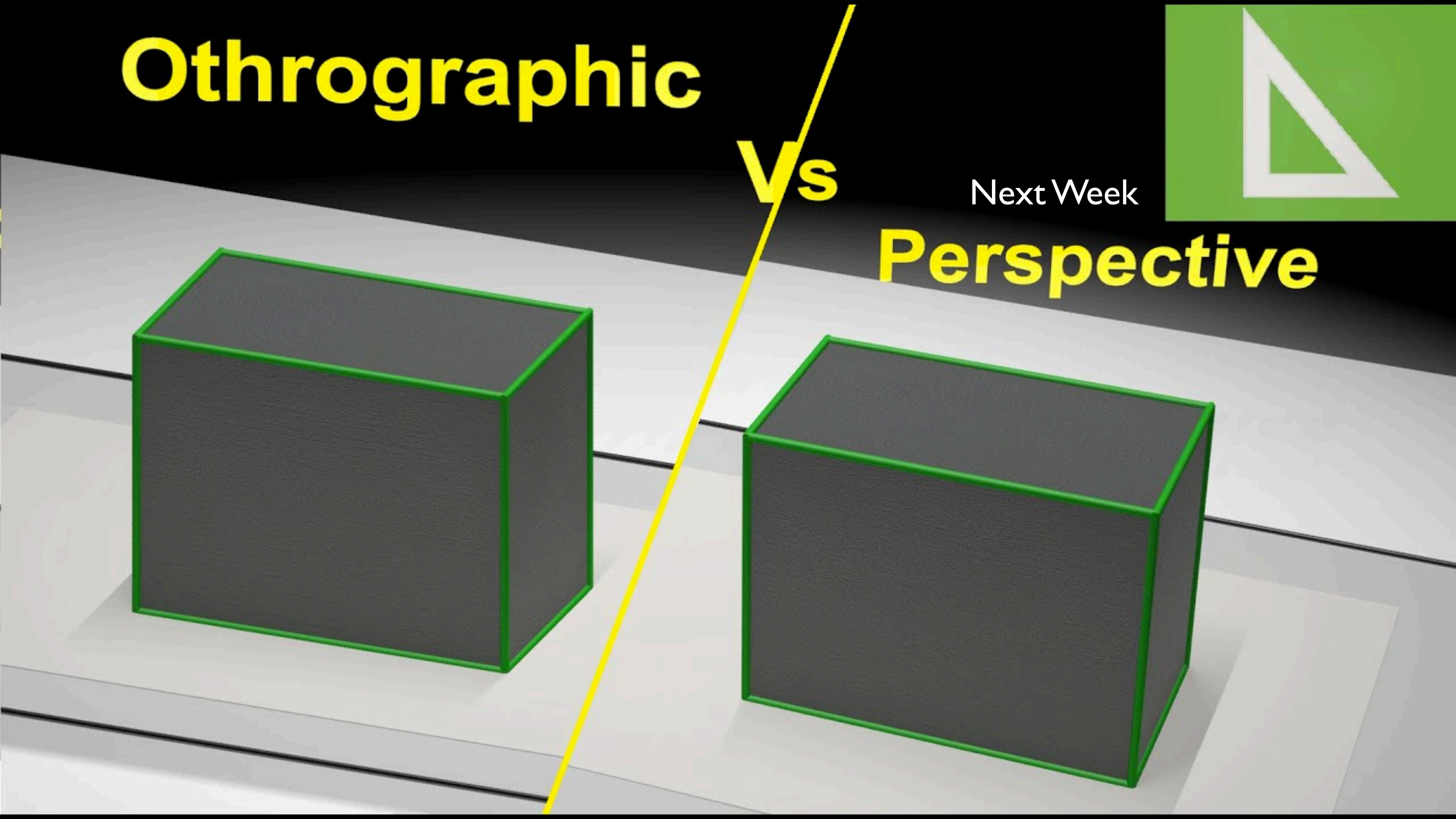
$$\mathbf{M}_{\mathrm{cam}} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Isometric drawing (using Orthographic projection)



Drawing1.dwg
Type a keyword or phrase
Sign In
File Edit View Insert Format Tools Draw Dimension Modify Parametric Window Help Express
Start
Drawing1*
[−][Top][2D Wireframe]
50
30
47
Ø12
110
R12
10
49
38
62
12
34
22
60
25
R12
12
50
30
12
110
22
34
N
W
TOP
E
S
WCS
Y
S
Model
Layout1
Layout2
61.3503, -83.1099, 0.0000
MODEL
1:1 / 100%
Decimal


Othrographic
Vs
Perspective
Next Week