Score generation with L−systems

Przemyslaw Prusinkiewicz

Abstract

A new method for algorithmically generating musical scores is presented and illustrated with examples. The idea is to produce a string of symbols using an L−system, and to interpret this string as a sequence of notes. The proposed musical interpretation of L−systems is closely related to their graphical interpretation, which in turn associates L−systems to fractals.

Reference

P. Prusinkiewicz: Score generation with L−systems. *Proceedings of the 1986 International Computer Music Conference*, pp. 455−457.

SCORE GENERATION WITH L-SYSTEMS

Przemyslaw Prusinkiewicz

Department of Computer Science
University of Regina
Regina, Saskatchewan, Canada S4S 0A2

ABSTRACT.

A new method for algorithmically generating musical scores is presented and illustrated with examples. The idea is to produce a string of symbols using an L-system, and to interpret this string as a sequence of notes. The proposed musical interpretation of L-systems is closely related to their graphical interpretation, which in turn associates L-systems to fractals.

KEYWORDS: generative modeling of music, L-systems, turtle geometry, fractals.

1. INTRODUCTION

The concept of parallel production systems, or L-systems, was introduced by Lindenmayer (1968). L-systems generate strings of symbols by repetitively substituting predecessors of given productions by their successors. There is, however, an essential difference between L-systems and the more widely known Chomsky grammars. In Chomsky grammars productions are applied sequentially, one at a time, while in the case of L-systems productions are applied concurrently to *all* symbols in a given string.

L-systems were conceived to formally describe the growth process of living organisms and in this context they were extensively studied by biologists and theoretical computer scientists. See Salomaa (1973), Herman & Rozenberg (1975) and Lindenmayer & Rozenberg (1976) for classic references. Another area of applications of L-systems was discovered by Smith (1984) who applied them to generate realistic images of plants and trees for computer imagery purposes. Recently I have shown that graphical applications of L-systems were not limited to plants and trees, but also included a wide range of fractal curves (Prusinkiewicz 1986). L-systems were also applied to generate fractals by Szilard & Quinton (1979) and Siromoney & Subramanian (1983).

This paper presents an application of L-systems to the algorithmic generation of musical scores. To this end, strings of symbols produced by an L-system are given a musical interpretation. The musical and graphical interpretations of L-systems are closely related. Because of the fractal nature of the created figures, the described approach is related to the work of Dodge & Bahn (1986) and Voss & Clarke (1978).

The necessary definitions related to L-systems are collected in Section 2. Section 3 applies the notion of a "turtle" (Papert 1980, Abelson & diSessa 1982) to interpret strings graphically, and presents examples of pictures generated by L-systems. Section 4 introduces a musical interpretation of L-systems. Section 5 describes the hardware and software used for experimentation. Section 6 presents conclusions and lists several possible extensions of the described model.

2. 0L-SYSTEMS.

This section summarizes the fundamental definitions and notations related to the simplest type of L-systems, called 0L-systems.

Let V denote an alphabet, V^* - the set of all words (strings) over V, and V^+ - the set of all nonempty words over V.

Definition 2.1. A **0L-system** is an ordered triplet $G = < V, \omega, P >$ where V is the **alphabet** of the system, $\omega \in V^+$ is a nonempty word called the **axiom** and $P \subset V \times V^*$ is a finite **set of productions**. If a pair (a, χ) is a production, we write $a \rightarrow \chi$. The letter a and the word χ are called the **predecessor** and the **successor** of this production, respectively. It is assumed that for any letter $a \in V$, there is at least one word $\chi \in V^*$ such that $a \rightarrow \chi$. A 0L-system is **deterministic** iff for each $a \in V$ there is exactly one $\chi \in V^*$ such that $a \rightarrow \chi$.

Definition 3.2. Let $G = < V, \omega, P >$ be a 0L-system, and suppose that $\mu = a_1 ... a_m$ is an arbitrary word over V. We will say that the word $\nu = \chi_1 ... \chi_m \in V^*$ is **directly derived** from (or **generated** by) μ and write $\mu \Rightarrow \nu$ iff $a_i \rightarrow \chi_i$ for all $i = 1,...,m$. A word ν is generated by G in a derivation of **length** n if there exists a sequence of words $\mu_0, \mu_1, ..., \mu_n$ such that $\mu_0 = \omega$, $\mu_n = \nu$ and $\mu_0 \Rightarrow \mu_1 \Rightarrow ... \Rightarrow \mu_n$.

Example. Consider an L-system with alphabet $V = \{a,b\}$, axiom $\omega = b$, and two productions: $a \rightarrow ab$ and $b \rightarrow a$. The words obtained by derivations of length 1, 2, 3, 4 and 5 are equal to *a*, *ab*, *aba*, *abaab* and *abaababa*, respectively (Fig. 1).

3. GRAPHICAL INTERPRETATION OF L-SYSTEMS.

This section formalizes the notion of the turtle interpretation of a word and provides examples of pictures generated by 0L-systems under this interpretation.

Definition 3.1. A **state** of the turtle is a triplet (x, y, α), where the Cartesian coordinates (x, y) represent the turtle's **position**, and angle α, called the turtle's **heading**, is interpreted as the direction in which the turtle is facing. Given the

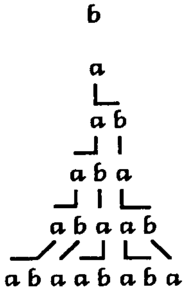
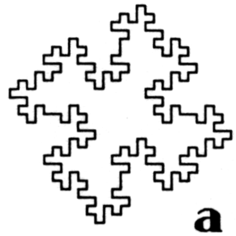
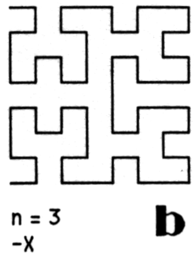


Fig. 1. Example of a derivation in an L-system.



n = 2
F+F+F+F
F ⟶ F+F-F-FF+F+F-F
+ ⟶ + - ⟶ -



n = 3
-X
X ⟶ -YF+XFX+FY-
Y ⟶ +XF-YFY-FX+
F ⟶ F
+ ⟶ + - ⟶ -

Fig. 2. Examples of curves generated by OL-systems under turtle interpretation.
(a) Quadratic Koch curve (Mandelbrot 1982).
(b) Hilbert curve (Hilbert 1891).

step size d and the **angle increment** δ, the turtle can respond to commands represented by the following symbols:

F Move forward a step of length d. The state of the turtle changes to (x', y', α), where $x' = x + d \cos\alpha$ and $y' = y + d \sin\alpha$. A line segment between points (x, y) and (x', y') is drawn.

f Move forward a step of length d without drawing a line.

+ Turn right by angle δ. The next state of the turtle is $(x, y, \alpha+\delta)$. (Here we assume that the positive orientation of angles is clockwise.)

– Turn left by angle δ. The next state of the turtle is $(x, y, \alpha-\delta)$.

All other symbols are ignored by the turtle (the turtle preserves its current state).

Definition 3.2. Let ν be a string, (x_0, y_0, α_0) - the initial state of the turtle, and d, δ - fixed parameters. The picture (set of lines) drawn by the turtle responding to the string ν is called the **turtle interpretation** of ν.

Example. Figure 2 presents turtle interpretations of the words generated by two deterministic 0L-systems. The angle increment δ is equal to 90°. The length of derivation n, the axiom ω of the 0L-system and the set of productions P is indicated for both pictures. Note that in the case of the Hilbert curve (Fig. 2b), the alphabet V contains auxiliary symbols X and Y. These are ignored when interpreting the final word ν but are necessary during its generation.

4. MUSICAL INTERPRETATION OF L-SYSTEMS.

Suppose that the Hilbert curve is traversed in the direction indicated by the arrow (Fig. 3) and the consecutive horizontal line segments are interpreted as notes. The pitch of each note corresponds to the y-coordinate of the segment, and the note duration is proportional to the segment length. The resulting sequence of notes forms a simple score shown in Figs. 4 and 5. Naturally, any curve consisting of horizontal and vertical segments can be interpreted in a similar way.

In the above example it is assumed that the notes belong to the C major scale and the first note is C. In general it is convenient to use a lookup table which allows for specifying an arbitrary mapping of y coordinates into note pitches.

5. IMPLEMENTATION.

From the hardware perspective, the experimental system consists of a Macintosh microcomputer controlling a DX-7 synthesizer. The software contains three modules. The **string generator** reads a file describing a given L-system, and produces the resulting string of symbols. This string is subsequently transferred to the **graphical interpreter** which draws the corresponding curve on the Macintosh screen. Information about consecutive line segments is passed to the **musical interpreter** which controls the synthesizer. Graphical and musical interpretations are synchronized, which means that the notes are played while the corresponding line segments are drawn. This effect is aesthetically pleasing (as well as useful for debugging).

6. CONCLUSIONS.

This paper presents a technique for generating musical scores, which consists of three steps:

- A string of symbols is generated by an L-system,
- This string is interpreted graphically as a sequence of commands controlling a turtle,
- The resulting figure is interpreted musically, as a sequence of notes with pitch and duration determined by the coordinates of the figure segments.

The scores generated using the above method are quite interesting. They are relatively complex (in spite of the simplicity of the underlying productions) but they also have a legible internal structure (they do not make the impression of sounds accidentally put together). A feature called **data base amplification** (the capability of producing complex objects using a small set of simple production rules (Smith 1984)) is as attractive in the musical applications of L-systems as it is in their applications in computer graphics.

Many extensions of the basic approach described in this paper are possible. In addition to the pitch and duration, string symbols generated by an L-system may control other

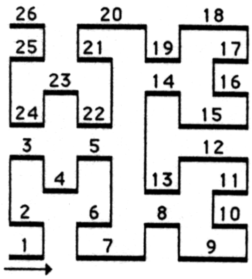


Fig. 3. Traversing the Hilbert curve.

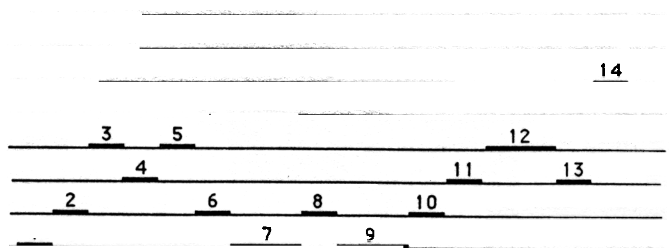


Fig. 4. The score associated with the Hilbert curve in the piano-roll notation.

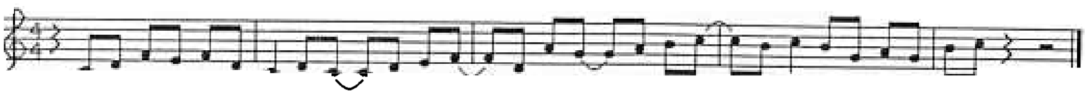


Fig. 5. The score associated with the Hilbert curv in the common musical notation.

parameters such as volume and tempo of the performance. L-systems can also generate polyphonic scores by assigning a musical interpretation to lines with branches. There is a standard technique for representing branching lines using L-systems (Prusinkiewicz 1986).

Score generation using L-systems is particularly attractive because it addresses several aspects of computer arts: the syntactic approach to musical composition, the use of fractals, and the combination of visual and aural aspects of a performance.

ACKNOWLEDGMENT.

The research described in this paper has been supported by grant number A0324 from the Natural Sciences and Engineering Research Council of Canada. This support is gratefully acknowledged.

REFERENCES.

ABELSON, H. & DISESSA, A.A. (1982). *Turtle geometry.* M.I.T. Press, Cambridge and London.

DODGES, C. & BAHN, C.R. (1986). Musical fractals. *Byte* **11**, Nr. 6, pp. 185-196.

HERMAN, G.T. & ROZENBERG, G. (1975). *Developmental systems and languages.* North-Holland, Amsterdam and Oxford.

HILBERT, D. (1891). Ueber stetige Abbildung einer Linie auf ein Flächenstück. *Math. Annln.* 38, pp. 459-460.

LINDENMAYER, A. (1968). Mathematical models for cellular interaction in development, Parts I and II. *Journal of Theoretical Biology* **18**, pp. 280-315.

LINDENMAYER, A. & ROZENBERG, G. (Eds.) (1976). *Automata, languages, development.* North-Holland, Amsterdam - New York - Oxford.

MANDELBROT, B.B. (1982). *The fractal geometry of nature.* W.H. Freeman, San Francisco.

PAPERT, S. (1980). *Mindstorms: children, computers, and powerful ideas.* Basic Books, New York.

PRUSINKIEWICZ, P. (1986). Graphical applications of L-systems. *Proceedings of Graphics Interface '86*, pp. 247-253.

SALOMAA, A. (1973). *Formal languages.* Academic Press, New York and London.

SIROMONEY, R. & SUBRAMANIAN, K.G. (1983). Space-filling curves and infinite graphs. In H. Ehrig, M. Nagl & G. Rozenberg (Eds.): *Graph grammars and their application to computer science*, Springer-Verlag, Berlin - Heidelberg - New York - Tokyo.

SMITH, A.R. (1984). Plants, fractals, and formal languages. *Computer Graphics* **18**, Nr. 3, pp. 1-10.

SZILARD, A.L. & QUINTON, R.E. (1979). An interpretation for DOL systems by computer graphics. *The Science Terrapin* **4**, pp. 8-13.

VOSS, R.F. & CLARKE, J. (1978). 1/f noise in music: music from 1/f noise. *Journal of the Acoustical Society of America* **63**, Nr. 1.