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Improving the Process of Plant Modeling:

The L+C Modeling Language

by

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Abstract

In this thesis I present the modeling language L+C. L+C is a language based on the formalism of L-systems. It has been created to address the need for a formalism that would allow the expression of complex plant models. Current plant models require the components of the model (organs or cells) to include many parameters to describe the state of the model. Also the need to express complex calculations has been addressed.

Signal propagation has been traditionally expressed using context-sensitive L-systems. L+C extends the formalism of L-systems by introducing new concepts: derivation direction and new context. These two concepts are the foundation of a new method of propagating signals in plant models: fast information transfer. Fast information transfer is an alternative, faster method of propagating signals in linear and branching structures represented by L-system strings.

The L+C modeling language is implemented in a plant modeling program lpfg, which together with cpfg (another L-system-based modeling program developed at the University of Calgary) are the core part of the modeling environment L-studio.

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1. Introduction

1.1. Motivation and scope of work

Since their introduction in 1968 by Aristid Lindenmayer [Lin1968], L-systems have evolved from a mathematical formalism into a modeling language. Initially L-systems were designed to express development of multi-cellular organisms at the level of individual cells. One of the early applications was a simple model of a bacteria filament [Lin1968]. In addition to linear structures, branching structures could be modeled using bracketed string notation [Lin1968].

As L-systems became more expressive the models became more complex. The growing complexity of the models put new demands on the expressive power of the L-systems. This reciprocal interaction has been developing and L-systems have become rich in elements that make it possible to develop models controlled by lineage [Lin1968, Lin1971], signals [Lin1968], allocation of resources [Pru1997a] and interaction with the environment [Mec1996]. The ability to integrate both plant growth and physiological processes allows simulation of plant development with accuracy and fidelity to mechanisms that can be observed in nature [Pru1990, Mec1996] and has led to scientifically valuable models which Room et al. have called *virtual plants* [Roo1996].

The main motivation for my research is a need for a formalism that will enable expression of more complex models, which can capture more phenomena, include additional elements and consider new factors to produce scientifically valid and interesting results. The experience gathered by scientists who model of plants shows that there is a need to define a common modeling platform. This platform will serve as a basis for further development of the modeling methodology. The research presented in my thesis is designed to address the L-systems part of this modeling platform.

L-systems as a modeling formalism can capture a class of dynamic systems with dynamic structures [Gia1997]. A dynamic system means that the quantitative information associated with the model, or elements of the model can change over time. For example in a plant structure leaves change in size and area, branches grow longer etc. In addition, the

structure itself can change: apices produce new branches; some branches may die and fall off.

The focus of my research was to extend the framework of L-systems to address the growing needs of the ever more complex functional-structural models. To address these needs I have extended some concepts present in L-systems as a formalism, added concepts known from other programming languages, and introduced new ones, not found in other formalisms.

Specifically the notion of parametric L-systems has been extended. Originally parametric L-systems allowed any number of numerical parameters to be associated with modules. This has been extended to allow parameters of any type. In particular, a parameter can be a user-defined structure. To address the need to express complex algorithms and calculations I have added user-defined functions, a concept common in other programming-languages.

An extension that is not found in other formalisms is the concept of *fast information transfer*. L-systems have supported information transfer using context-sensitive productions. This is a universal method for transferring any type of information: the propagation of hormones, nutrients and other signals through the plant. An inherent feature of this method is that the number of simulation steps required to transfer information from point A to point B is proportional to the distance between these points measured as the number of modules between them. This feature becomes a limitation when the speed of signal propagation is high compared to the growth rate of the structure (for example propagation of forces and torques in biomechanical models). Fast information transfer removes this limitation making it possible to transfer signals throughout the whole structure represented by the L-system string in one simulation step.

The number and weight of extensions postulated in my research made it justified to design a completely new modeling language. The new language would combine elements from L-systems and an imperative programming language. For the elements known from general-purpose programming, such as functions and user-defined data types (structures), the syntax from C++ was chosen. Rather than extending a modeling language with more programming elements, my approach is to add L-system elements to C++. Prusinkiewicz

and Hanan [Pru1992] presented a similar idea for adding L-systems to C. Their work was limited to context-free L-systems, without parameters.

By adding elements of L-systems to C++ the whole power of C++ can be used to describe algorithms and data structures required for a model. Yet the introduction of L-system constructs changes the structure of the language: the models are declarative in nature and consist of productions. Therefore the language has been renamed L+C.

In addition to the language, I have designed and implemented the modeling program *lpfg*. It executes models specified in the L+C modeling language. The results can be rendered as a three-dimensional visualization of the model or stored in an external file for further analysis. I have also created a comprehensive modeling system, L-studio. L-studio was inspired by the *vlab* modeling environment originally created by Mercer [Mer1990, Mer1991] and then extended by Federl [Fed1999]. L-studio combines the functionality of several programs to create and render complex models, and is compatible with vlab. The system has proved to be useful for biologists (it is currently being used in approximately 100 locations worldwide).

1.2. Organization of the dissertation

This section outlines the organization of the thesis.

Chapter 2 presents the history of L-systems, including the main concepts, definitions and applications that are essential to understanding my research. In chapter 3 the new concepts that I have added to L-systems are presented. The new modeling language created to include these concepts is presented in chapter 4. Chapter 5 is devoted to some considerations on how to implement the language and internal representation of the L-system string. Chapter 6 discusses the interface which is used to communicate between the modeling program (*lpfg*) and the translated L+C model. Chapter 7 contains examples that demonstrate the benefits of using L+C over traditional L-system language.

Chapter 8 describes the L-studio plant modeling environment which I have created in the scope of my research. There were two principal reasons for creating this environment. First, there were several ideas related to interactive and visual modeling techniques that could be tested. In addition there was considerable interest in a system that would work in the MS Windows environment coming from biologists who use L-system-based simulations as one

of their research tools. The description of L-studio emphasizes the interactive and visual modeling concepts I have introduced or extended.

The conclusions, including a summary of contributions and issues for further research are presented in chapter 9. Appendix A contains the user's manual of *lpfg* modeling program, based on L+C modeling language.

1.3. Document conventions

Source code listings are printed using `fixed-width font`.

2. Lindenmayer systems

Lindenmayer systems (or L-systems) are a mathematical formalism introduced by Aristid Lindenmayer in 1968 [Lin1968] to model multi-cellular organisms. Ability to express branching structures (with bracketed strings) made L-systems particularly useful in modeling plants. The following sections describe the main concepts of L-systems.

2.1. D0L-systems

The simplest type of L-systems are D0L-systems: **D**eterministic, context-free (**0**), **L**-systems. The formal definition of D0L-systems is given below (based on the definition given in [Roz1980]):

- An alphabet is a finite set of *letters* denoted as V. The letters are also called modules.
- A *word* is a sequence of letters over an alphabet. The set of all words over alphabet V is denoted as V^*.
- A *production* is a pair (a, u) denoted as $a \to u$, where a is a letter and u is a word. a is called predecessor and u is called successor.
- A D0L-system is a triplet $G = \langle V, \omega, P \rangle$, where V is an *alphabet*, $\omega \in V^*$ is a word called the *axiom*, and P is a *set of productions* such that $\forall a \in V : \overset{1}{\exists} p_a \in P$, where p_a denotes a production that has module a as its predecessor.

Production $a \to u$ is said to match module a. By convention it is assumed that if no production is specified for a module a explicitly then the identity production $(a \to a)$ is added implicitly. A production can also specify that the current module should be *removed* from the string. This is expressed by specifying an asterisk (*) or ε (in formal notation) as the successor.

The process of applying productions and creating a new string is called *string rewriting*. Execution of an L-system consists of a series of string rewritings, which are then called *derivation steps*. In L-system rewriting the productions are applied in parallel to all

modules in the string. The productions in L-systems are sometimes labelled (p_n:) for presentation or discussion purposes, but the labels do not appear in the actual code.

A classical example of a D0L-system describes the growth of the vegetative segment of *Anabaena catenula* [DeK1987, Lin1971]. A vegetative segment consists of cells that can be in one of two states – young (shorter) or ready to divide (longer). The cells also have two possible polarities. The polarity specifies which of the daughter cells will be shorter. In the following L-system (reproduced after [Pru1990]) the letters *a* and *b* specify the two states and the subscripts *l* and *r* specify cell polarities.

Listing 1 Development of *Anabaena catenula* filament.

axiom: a_r

p_1: $a_r \rightarrow a_l b_r$

p_2: $a_l \rightarrow b_l a_r$

p_3: $b_r \rightarrow a_r$

p_4: $b_l \rightarrow a_l$

The developmental sequence determined by the L-system in Listing 1 begins with the axiom a_r, and produces a new word with each derivation step:

$$a_r$$

$$a_l b_r$$

$$b_l a_r a_r$$

$$a_l a_l b_r a_l b_r$$

$$b_l a_r b_l a_r a_r b_l a_r a_r$$

2.2. Bracketed L-systems

To represent a branching structure using a string of letters (which by definition is a linear structure), two reserved modules were introduced as a part of the original definition of L-systems [Lin1968]. These modules are the left bracket ([) and the right bracket (]) and they specify the beginning and the end of a branch, respectively.

The following L-system generates a simple branching structure (after [Pru1990], p. 25) consisting of two types of modules: apices (A) and internodes (I):

Listing 2 L-system generating simple branching structure

```
axiom: A

A → I[A][A]IA
I → II
```

Figure 1 visualizes the structures generated by the L-system in Listing 2 with both modules visualized as straight lines of unit length. Modules `A` (apex) are drawn in green, `I` (internode) modules are drawn in black. Lateral branches are rotated relative to the parent branch. The images are scaled to the same size to better visualize the growing complexity of the structure.

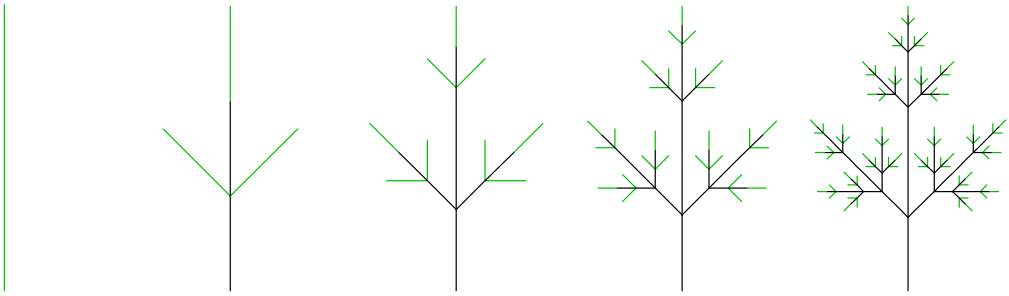


Figure 1 Branching structure generated by L-system in Listing 2

2.3. Graphical representation of L-systems

The desire to represent plant structures graphically led to new interpretations of L-systems. For example, the L-system in Listing 2 does not specify the directions or angles between the parent branch and lateral branches. Yet branch orientation is a fundamental feature of plants. In Lindenmayer's early work [Lin1971], lateral branches were drawn as alternately left and right. Hogeweg and Hesper [Hog1974] represented geometric aspects (branching angles, length of branches) according to externally defined rules. This concept was later extended to include 3D structures [Smi1984].

The most common interpretation used today is based on the LOGO-style turtle [Abe1982], as introduced by Prusinkiewicz [Pru1986]. The main concept is that some modules in the L-system string are interpreted as commands executed by a turtle. In 2D the *state of the turtle* is characterized by its position and orientation. A vector called the heading vector specifies the orientation. Basic commands executed by the turtle are:

- F – move forward one unit in the direction specified by the heading vector and draw a line
- f – move forward one unit in the direction specified by the heading vector without drawing a line
- + (plus), – (minus) rotate left, right around the position by a predefined angle.

To represent three-dimensional structures the state of the LOGO-style turtle has been extended. In the 3D systems the orientation of the turtle is defined by three mutually perpendicular vectors called heading, left and up.

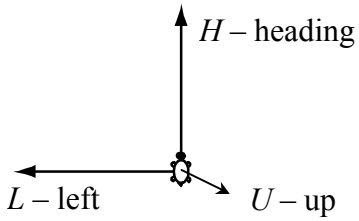


Figure 2 Turtle orientation defined by vectors *H L* and *U* (pointing to the viewer)

The L-system presented below (after [Pru1990]) generates the Koch snowflake using basic turtle commands. The rotation angle associated with the rotate commands + and – is specified externally to be 60 degrees.

Listing 3 L-system generating Koch snowflake

```
Axiom: F--F--F
F → F+F--F+F
```

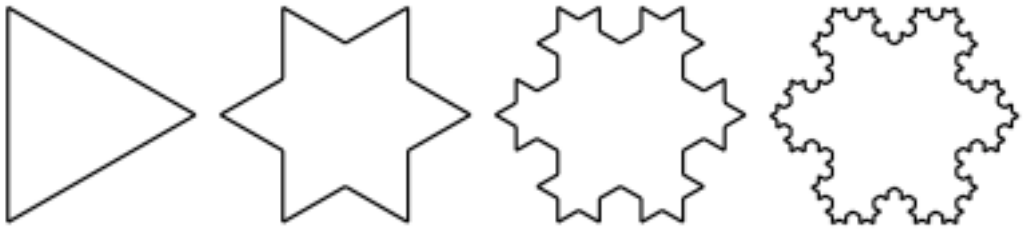


Figure 3 Koch snowflake generated by the L-system in Listing 3

Additional commands were also introduced [Pru1986] to make it possible to generate three-dimensional structures. These commands are

- rotation around the left vector (^ – pitch up, & – pitch down)
- rotation around the heading vector (\ – roll left, / – roll right).

When drawing branching structures specified by bracketed L-systems, the modules [and] are interpreted as follows:

-] – the turtle state is pushed on a stack
- [– the turtle state is popped from the stack.

In addition to the position and orientation the turtle state can also include drawing parameters, such as drawing colour, line width etc.

2.4. Parametric L-systems

D0L-systems, as presented in the previous sections, can represent qualitative information in which each type of module represents a different type of components in the model, such as a cells or organ. Some quantitative information (such as the length of internodes or the magnitude of angles of rotation) can also be specified by the D0L formalism using multiple modules to express different lengths of lines or rotation angles. For example in Listing 3 the two modules + or – are used to represent a rotation of 120 degrees left and right respectively.

However it is impossible to express such a simple figure as an isosceles right triangle where the line lengths do not have a common denominator. This limitation has been addressed by parametric L-systems [Pru1990, Han1992].

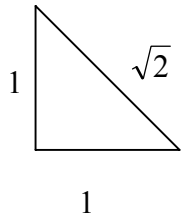


Figure 4 Isosceles right triangle

The essence of parametric L-systems is that each module consists of a symbol together with associated numerical parameters. In the productions parameter values are referred to using formal parameters. Additionally, formal parameters can be used in arithmetic expressions. The expressions can be used to calculate new values of parameters in the production's successors.

The formal definition of parametric D0L-systems is as follows (after [Pru1990]):

- V is the alphabet.
- $\sum$ is the set of formal parameters.
- $C(\sum)$ is the set of all logical expressions with parameters from $\sum$.
- $E(\sum)$ is the set of all arithmetic expressions with parameters from $\sum$.
- $\omega \in (V \times \Re^*)^+$ is a nonempty parametric word called the axiom.
- P is a finite set of ordered productions of the form *pred*: $cond_{opt}$ → *succ* such that $pred \in V \times \sum^*$, $cond \in C(\sum)$ and $succ \in (V \times E(\sum)^*)^*$. The components of productions are called predecessor, condition and successor respectively. If the condition *cond* is omitted in a production it is assumed to evaluate to true.

In the case of parametric L-systems, the process of matching productions during string rewriting is more complex than for D0L-systems. For a production to match a module in the string, the following conditions must be met:

1) The module's letter must match the letter in the predecessor,
2) The number of actual parameters associated with the module and the number of formal parameters in the predecessor must be the same,
3) The condition *cond* must evaluate to true.

For example, production

```
A(t) : t>5 → B(t+1)A(t/2)
```

can be applied to module `A(6)` and will produce parametric word consisting of two modules: `B(7)A(3)`.

2.5. Context-sensitive L-systems

In context-free L-systems productions are applied regardless of the context in which the predecessor module appears. Context-sensitive L-systems make it possible to specify what modules must be in the neighbourhood of the modules being replaced for the production to be applied. Context-sensitive L-systems are necessary to express information flow in the modelled structure. For example the transfer of nutrients or hormones throughout a plant

structure can be modelled using context-sensitive L-systems [Lin1968]. Context-sensitive productions have the form:

$$lc < pred > rc : cond \rightarrow successor$$

The symbols < and > separate the three components of the predecessor: the left context (*lc*), the strict predecessor (*pred*), and the right context (*rc*).

The process of matching productions in context-sensitive L-systems is governed by a set of rules that are discussed in the following section.

2.5.1. How the productions are matched

When rewriting the string it is necessary to determine which production must be applied to each module in the string. The process of determining the applicable production is called *production matching*. For every module in the string, productions are checked for matching. The productions are checked in the order in which they are specified in the L-system.

For a production to match, all three components of the predecessor (left context, strict predecessor and right context) must match. The rules for matching each of these components are different. This is because the L-system string is a means of representing branching structures and symmetric operations on the string do not (in general) correspond to symmetric operations on the branching structure. No good definition of context in branching structure can be found in the L-systems literature. One of partial definition is the work by Prusinkiewicz et al. [Pru1988].

This section contains a detailed explanation of rules that control the process of production matching. Good understanding of these rules is necessary for proper understanding of the concept of fast information transfer in branching structures (described in 3.3.2).

When the strict predecessor is compared with the contents of the string in the current position in order for it to match the modules in the strict predecessor have to match exactly the modules in the string.

When matching the right context and a module in the context is not the same as module in the string the following rules apply:

- If a module in the string is [and the module expected is not [then the branch is skipped. This rule reflects the fact that modules may be topologically adjacent, even though in the string representation of the structure the two modules may be separated by modules representing the lateral branch `B` (see Figure 5).
- When a branch in the right context ends (with a right bracket) then the rest of the branch in the string is ignored by skipping to the first unmatched]. This rule also reflects the topology of the branching structure, not its string representation. For example in Figure 6, module `C` is closer to `A` than `D`.
- If multiple lateral branches start at a given branching point, then the predecessor in Figure 6 would check the first branch (see Figure 7). To skip a branch it is necessary to specify explicitly which branch at the branching point should be tested (see Figure 8). This notation is a simple consequence of the rule presented in Figure 6. In the current L-system notation there is no shortcut to specify the second, third etc. lateral branch in a branching point without explicitly including pairs [ ] in the production predecessor. There is also no way to specify "any of the lateral branches".

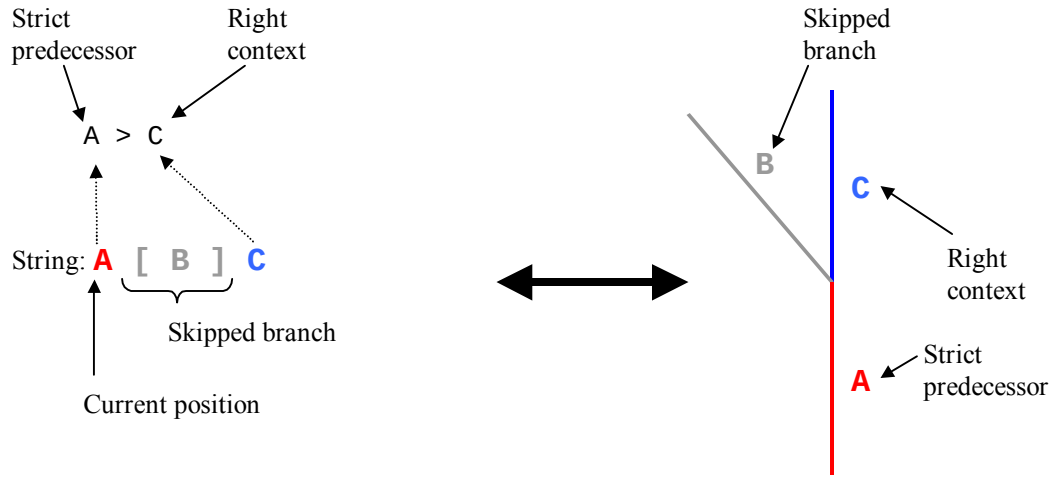


Figure 5 Matching right context, lateral branches are implicitly ignored

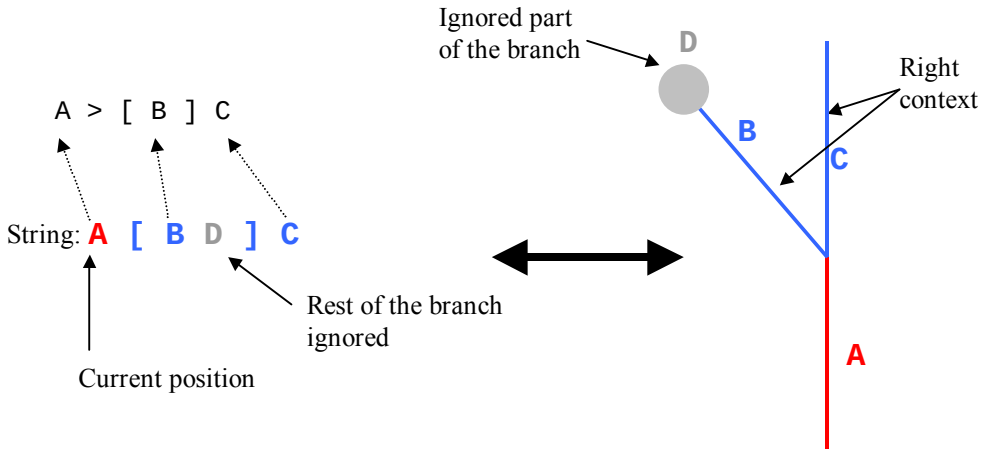


Figure 6 Matching right context, remainder of lateral branch is implicitly ignored

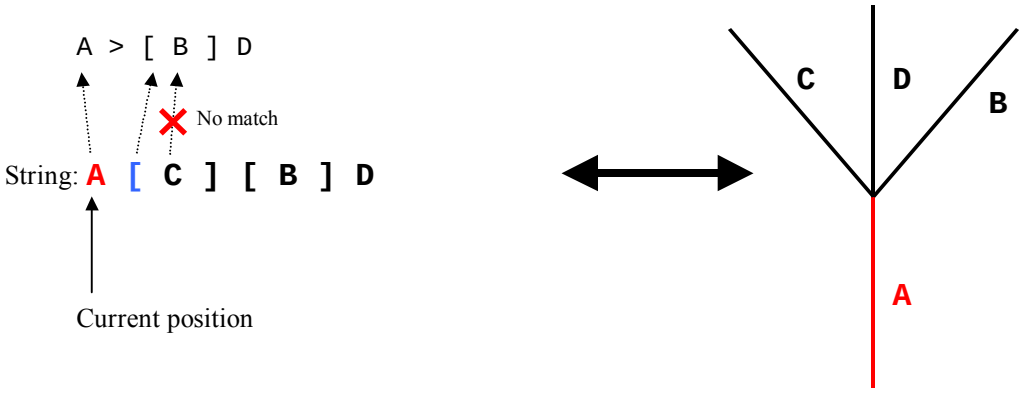


Figure 7 Problem with multiple lateral branches when matching the right context

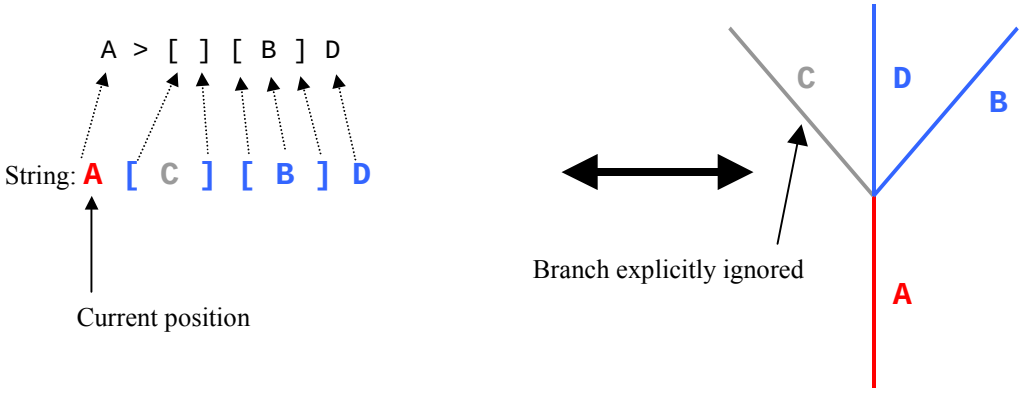


Figure 8 Explicit enumeration of lateral branches in the right context

When matching the left context the following rules apply:

- Module [is always skipped, since the preceding module will be topologically adjacent (see Figure 9).
- If the module indicates the end of a branch then the entire branch is skipped (Figure 10).

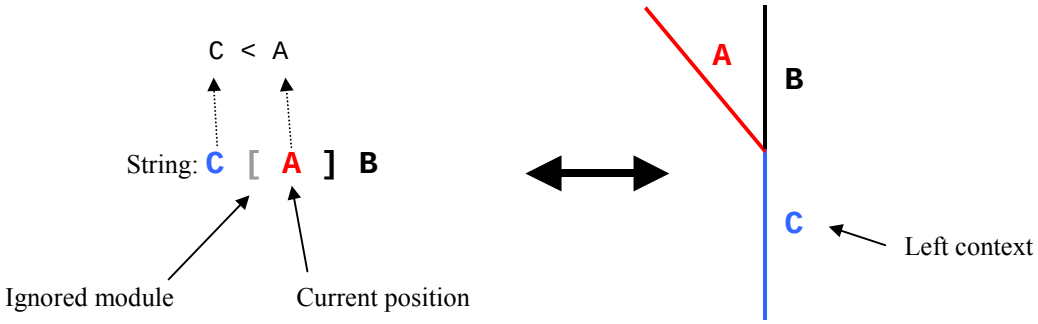


Figure 9 Matching left context, beginning of the branch implicitly ignored

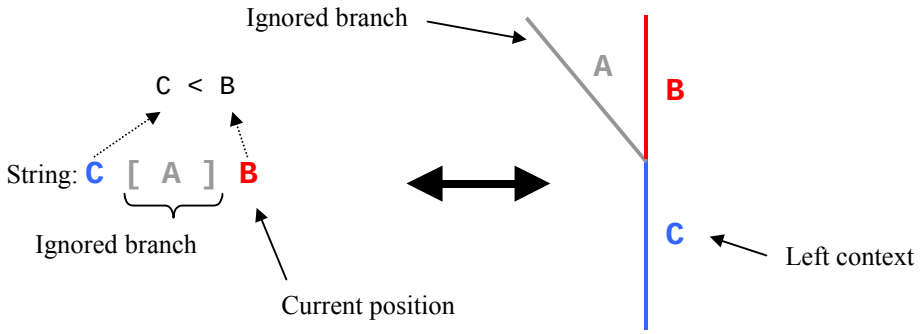


Figure 10 Matching left context, lateral branches implicitly ignored

The rule illustrated in Figure 9 is a pronounced manifestation of asymmetry in the left context – right context relationship: module `C` is left context of both `A` and `B`. But `C`'s right context is `B` (unless [ ] delimiters are used explicitly). The relation of the left context can be thought of as the *parent* module: the module before (below) the branching point. It is then natural to say that `C` is parent module for both `A` and `B`. The distinction between main branches and lateral branches can appear to be an implementation dependent artefact, but it actually can be biologically justified (see for example [Bor1984]).

2.5.2. Ignored and considered modules

The L-system presented below (Listing 4) describes the propagation of an acropetal signal using a context-sensitive production. This signal can be, for example, a hormone. `J` represents an internode where the hormone is present (red line), and `I` represents an internode where it is not present (black line).

Listing 4 Acropetal signal propagation implemented using context-sensitive L-system

```
Axiom: I[+J][-J]J[+J][-J]J[+J][-J]J
p1: I < J → I
p2: I+ < J → I
p3: I- < J → I
```

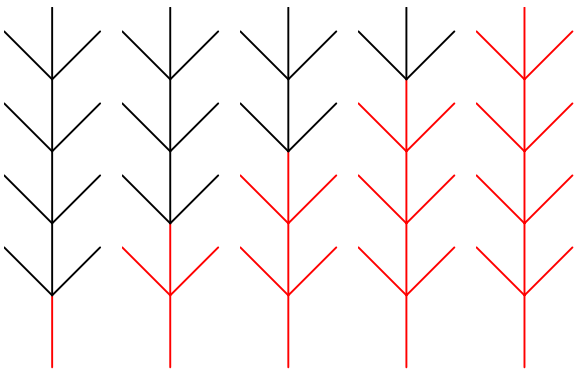


Figure 11 Propagation of acropetal signal – output from L-system in Listing 4

The L-system consists of three productions: p_1 is responsible for transferring the signal to the main branches; p_2 and p_3 are responsible for transferring the signal to the lateral branches. They are necessary because every `J` is preceded by a + or a – (the modules that specify rotation, see section 2.3) and the left context in p_1 doesn't match the sequence of modules `I[+J]` or `I[-J]`.

Productions p_2 and p_3 do not add any new information to the model. They have to be present because of the geometric properties of the model. To be able to abstract from such details, the notion of ignored modules was introduced. It makes it possible to specify a list of modules that are ignored when checking for matching context so that Listing 4 can be rewritten as follows:

Listing 5 Acropetal signal propagation implemented using the ignore statement.

```
ignore: +-
Axiom: I[+J][-J]J[+J][-J]J[+J][-J]J
I < J → I
```

If the list of ignored modules is long it may be more practical to list only the relevant modules that appear in the left or right context. This is done using the consider statement. Consequently Listing 4 can be then rewritten as follows:

Listing 6 Acropetal signal propagation implemented using the consider statement

```
consider: I
Axiom: I[+J][-J]J[+J][-J]J[+J][-J]J
I < J → I
```

In summary: the presence of ignored and considered modules adds two rules to the test for matching context.

- When the right context is checked, modules that are not to be considered (those listed after the `ignore` keyword or those *not* listed after the `consider` keyword) are skipped (Figure 12).
- Similarly, when checking the left context, ignored modules are skipped (Figure 13).

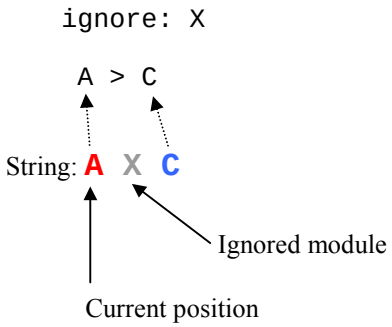


Figure 12 Matching right context with ignored modules

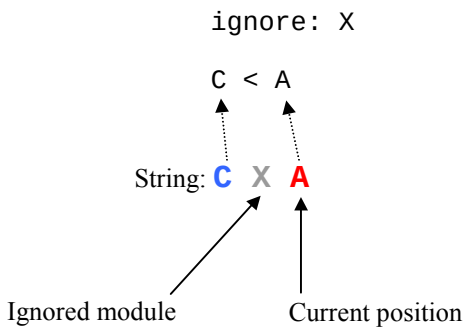


Figure 13 Matching left context with ignored modules

2.6. L-systems with programming statements

As the models created using parametric L-systems became more complex, Hanan [Han1992] extended L-systems to include some programming language constructs (see also [Pru1992, Pru1996]).

Programming constructs include:

- Assignment of variables (local and global),
- Calls to predefined functions,
- Conditional statements (`if … else`),
- Loops.

Global variables can be used to store global information about the model e.g. the number of leaves, flowers. Expressions used in parametric L-systems productions can be complicated and they are often used in more than one module within one production. Therefore, local temporary variables were introduced that could store a calculated value that could be used throughout a production.

```
A(n) → F[+A(n+1)][-A(n+1)]                    (1)
```

Hanan extended the cpfg language [Han1992] to include the following syntax for productions:

$$lc < pred > rc : \{ \alpha \}_{opt}\ cond\ \{ \beta \}_{opt} \rightarrow succ$$

where α and β are optional C-like compound statements, and *cond* is a logical expression. During the string rewriting if the production predecessor (strict predecessor, left context and right context) matches the current string position (see section 2.5.1) the statement α (if present) is executed and *cond* evaluated. Thus, the production (1) can be rewritten as:

```
A(n) : { new_n = n+1; } 1 → F[+A(new_n)][-A(new_n)]          (2)
```

If *cond* evaluates to true (non-zero) value then β (if present) is executed and the production applied (the successor added to the new string). But if *cond* evaluates to zero then the production is not applied. In this case the next production declared is tested for matching. This makes it possible to specify more than one production that has the same predecessor but produces different modules depending on the value of *cond*. The condition can depend on the global state of the model (global variables), local conditions (actual parameters of modules in the predecessor) or both.

Other elements added to the cpfg language by Hanan [Han1992] are predefined functions that include mathematical functions, pseudo-random number generators etc. They are used in computations or in file and console I/O operations (results of simulations can be stored in external files for further analysis using other programs or simply displayed in a console).

In addition to productions, programming statements can be used in *control statements*. Control statements are procedures, which are called during the execution of L-system program. There are four control statements.

`Start`: { *code* }	Executed at the beginning of the simulation
`StartEach`: { *code* }	Executed before each derivation step
`EndEach`: { *code* }	Executed after each derivation step
`End`: { *code* }	Executed at the end of the simulation

All control statements are optional.

Listing 7 presents an L-system program that creates a simple branching structure (with some randomness). The control statements are used to gather statistical data about the

model. The data are stored in an external file. Because the L-system source file is preprocessed using a standard C preprocessor, `#define` is used to define constants.

Listing 7 L-system with control statements and predefined functions

```
#define STEPS 50
#define MATURE 1
#define dt 0.2

derivation length: STEPS

Start: {
fp = fopen("output.dat", "w");
db = 0;
step = 0;
}

StartEach: {
ap = 0;
step = step+1;
}

EndEach: {
if (ap>0)
  { fprintf(fp, "%.0f apices created in step %.0f\n", ap, step); }
}

End: {
fprintf(fp, "Total: %.0f dead buds\n", db);
fclose(fp);
}

Axiom: A(0)

p1: A(t) : t<MATURE → A(t+dt)
p2: A(t) : ran(1)<0.8 { ap = ap+2; } → F(0.2)[+A(0)][-A(0)]
p3: A(t) : 1 { db = db+1; } → ,G(0.2);

p4: F(t) → F(1.08*t)
```

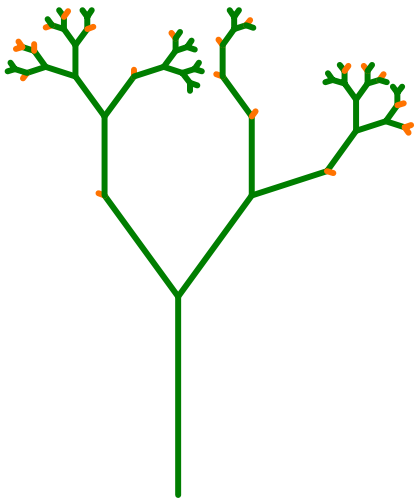


Figure 14 Sample image generated by the L-system from Listing 7

There are three global variables declared in the `Start` statement: `fp` (file pointer), `db` (dead buds counter) and `step` (step number). Before every derivation step global variable `ap` (apex counter) is set to 0 and `step` is incremented.

Initially the model consists of a young apex A(0). Production p_1 increases the age of the apex until it reaches the mature state (condition `t<MATURE`). When the apex is mature production p_2 is applied with a 0.8 probability (condition `ran(1)<0.8`, where `ran` is a pseudo-random number generator with uniform distribution). If applied, p_2 produces an internode (module `F`) and two lateral branches with young apices. It also increments the global variable `ap` by two. This variable stores the number of apices produced in every derivation step. If p_2 is not executed then production p_3 is applied. In that case the apex is replaced with a dead bud drawn in an alternate colour[1] and the global variable `db` is incremented. Production p_4 increases the internode length by a constant factor of 1.08. Thus internodes created earlier will be longer than those created later.

A sample output generated by the L-system from Listing 7 is given below.

```
2 apices in step 6
4 apices in step 12
6 apices in step 18
8 apices in step 24
12 apices in step 30
```

[1] In *cpfg* language modules , and ; change the current drawing colour.

```
22 apices in step 36
32 apices in step 42
46 apices in step 48
Total dead buds 21
```

2.7. Interpretation rules

When creating a plant model it is important to distinguish two elements in the process: the structure of the plant model and its visualization. For example, during the development and testing of a model, organs can be visualized simply: stems as straight lines, leaves as polygons, etc. Once the model generates the correct structure and topology, the visual aspect can be extended: lines can be replaced with cylinders, and polygons replaced with 3D surfaces (such as Beziér parametric surfaces). If these two aspects can be separated the model is clearer and easier to maintain. This goal was achieved by introducing homomorphisms for interpreting the string.

In formal language theory, a homomorphism defines a mapping from an alphabet V to words in another alphabet V_h [Roz1980]. Formal definition of non-parametric L-systems with homomorphisms is as follows (after [Pru1997]):

- V and V_h are two alphabets
- $G = \langle V, \omega, P \rangle$ is an L-system over alphabet V
- $h : V^* \rightarrow V_h^*$ is a homomorphism
- The ordered quintuplet $H = \langle V, V_h, \omega, P, h \rangle$ is an L-system with a homomorphism with the support G and homomorphism h.

Elements of h are called interpretation rules. Interpretation rules are applied only during the interpretation of the string (for example when visualizing the model[2]). These rules are *not* applied when deriving the string.

The syntax for interpretation rules is the same as productions, except that interpretation rules are always context-free. During interpretation, modules in the string are replaced with their image specified by the interpretation rules. By convention, if no interpretation rule is specified for a module then its image is the module itself. Interpretation rules are applied

[2] The string is also interpreted in other cases, for example see 2.9

recursively on the resulting words until the word contains only modules that are mapped into themselves (terminal symbols) or until a predefined recursion depth is reached.

The interpretation rules are more closely related to Chomsky grammars than L-systems. In Chomsky grammars the productions do not define development but the structure. Also, productions in L-systems are applied in parallel, whereas productions in Chomsky grammars are applied sequentially.

The following L-system is an extension of the program presented in Listing 2. It includes interpretation rules that specify how to draw the organs. An apex is visualized as a line and a circle[3], both drawn using an alternative colour (orange). Internodes are visualized as straight lines drawn using the default colour (green). In the *cpfg* language interpretation rules are preceded by the keyword `homomorphism`.

[3] In *cpfg* language `@o` draws a circle

Listing 8 L-system from Listing 2 with interpretation rules

```
#define STEPS 4
derivation length: STEPS
axiom: A
A → I[+A][-A]IA
I → II
homomorphism
A → ;F@o
I → F
```

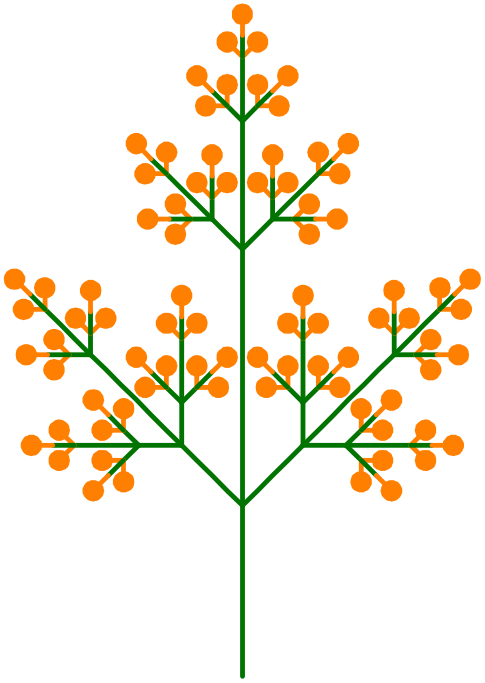


Figure 15 Image generated by the L-system presented in Listing 8

Figure 16 shows the developmental sequence of a model with interpretation rules. When the initial string μ_0 is visualized, the interpretation rules (h) map this string into the string v_0, which contains the graphical information. After the visualization a derivation step is performed (P) that applies the productions to the original string μ_0 and produce string μ_1. This string is again mapped using the interpretation rules into the string v_1 and so on.

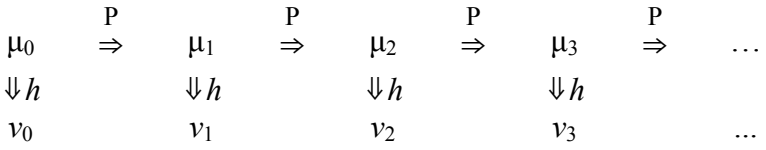


Figure 16 Developmental sequence of a model with interpretation rules.

Interpretation rules do not always have to be applied after every derivation step. In some cases for example the simulation is performed but only the final string is visualized.

2.8. Decomposition rules

Decomposition rules are formally and syntactically related to interpretation rules. Decomposition rules are context-free. They are also applied recursively. The two fundamental differences between decomposition and interpretation rules are that the successor of a decomposition rule is *inserted* into the string, and decomposition rules are always applied after each derivation step. Whereas the interpretation rules express the idea "module *looks* like this", decomposition rules express the idea "module *consists* of the following".

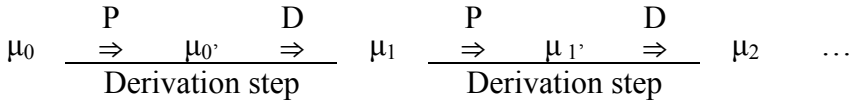


Figure 17 Developmental sequence of a model with decomposition rules.

Figure 17 shows the developmental sequence of a model with decomposition rules. First productions (P) are applied and the initial string μ_0 is replaced with string $\mu_{0'}$. Then the decomposition rules (D) are applied and produce string μ_1. The string $\mu_{0'}$ can be considered an intermediate state and the application of the decomposition rules can be thought of as a post-processing phase of the derivation step.

The L-system presented below implements a developmental model of a simple branching structure.

Listing 9 L-system with decomposition and interpretation rules

```
#define max_t 2
#define dt 0.4

derivation length: 30

Axiom: A(0)

p1: A(t) --> A(t+dt)
p2: I(t) --> I(t+dt)

decomposition

d1: A(t) : t>max_t --> I(max_t)[+A(t-max_t)][-A(t-max_t)]

homomorphism

i1: A(t) --> ;(1)F(t)@o(0.8)
i2: I(t) --> ;(2)F(t)
```

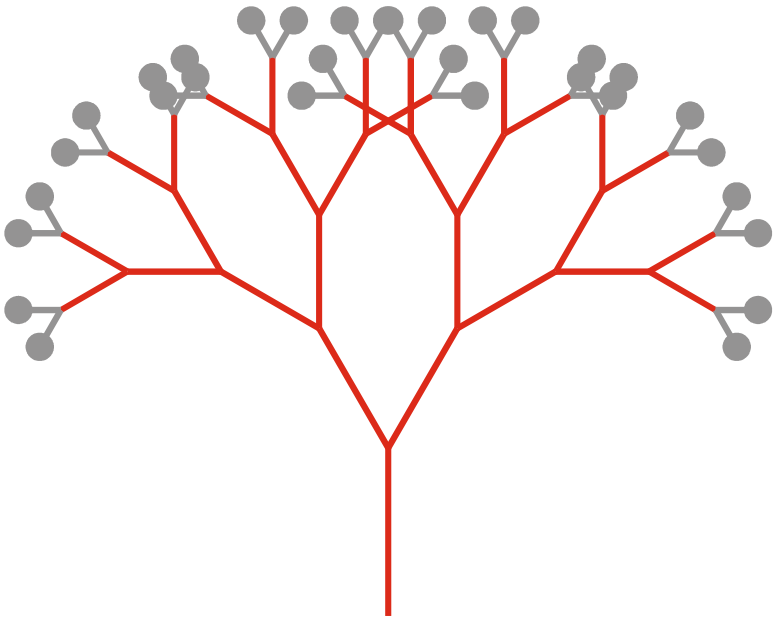
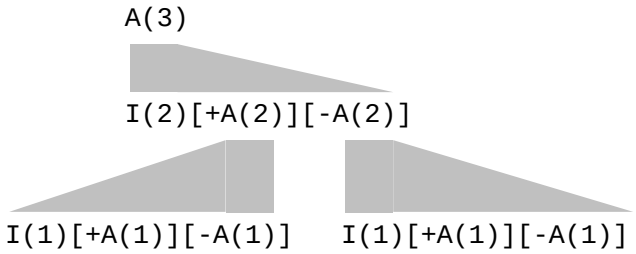


Figure 18 Structure generated by L-system in Listing 9

The model consists of two types of modules: A (apex) and I (internode). Both module types have one parameter, that represents the age. Initially the model consists of a young

apex A(0). The productions p_1 and p_2 advance time. The actual development is implemented in the decomposition rule d_1. This rule specifies that a mature apex produces an internode and two lateral apices. This decomposition rule is very similar to the production p_1 from the model presented in Listing 7. The main difference is that the age of the new apices (as well as the age of the iternode) is calculated as the difference between the current age of the apex and maximum age. This expresses the idea: if there is an apex of age `t` and `t>max_t` then this apex must have produced an internode and two apices `t-max_t` time ago. So the internode and the apices are already that old. This idea can be also expressed using a production. A production however will not produce correct results if `t-max_t>max_t`. For example if `max_t=1` then using a production string `A(3)` would be replaced with `I(2)[+A(2)][-A(2)]`. This is wrong because an apex cannot be older than 1, but because decomposition rules are applied recursively the string `A(3)` will be properly decomposed into:



effectively producing:

```
I(2)[+I(1)[+A(1)][-A(1)]][-I(1)[+A(1)][-A(1)]]
```

The fact that decomposition rules are applied recursively makes it possible to create for example a model of a tree, where every derivation step corresponds to a time step equal to several years, while branches are produced every year.

Another application of decomposition rules is to generate a sequence of the same (or similar) modules or groups of modules. For example the decomposition rule in Listing 10 generates `n` repetitions of the sequence `F @o(0.1)`.

Listing 10 Decomposition rule used to generate a sequence of modules

```
axiom: A(4)
decomposition
A(n) : n>0 → F @o(0.1) A(n-1)
```

After the string is initialized the module `A` is decomposed into pairs of modules `F` and `@o` followed by an `A`. The number of repetitions is specified by the value of the actual parameter of `A` in the axiom (see Figure 19).

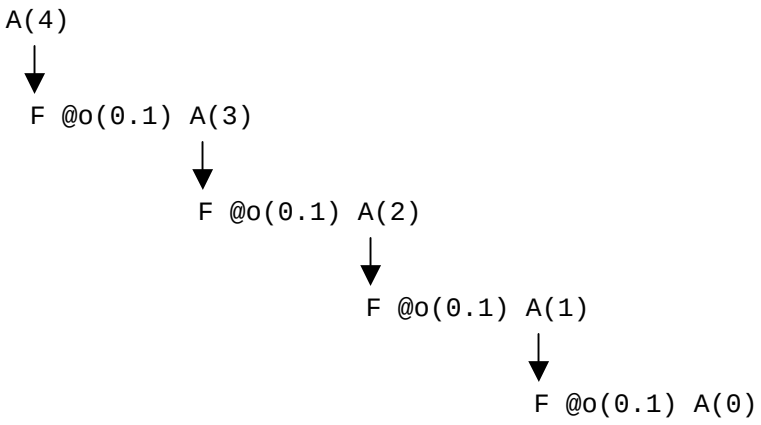


Figure 19 Decomposition rule applied recursively

Effectively the module `A(4)` is replaced by

```
F @o(0.1) F @o(0.1) F @o(0.1) F @o(0.1) A(0)
```

Another decomposition rule can be added to remove the trailing module `A(0)`:

```
A(n) : n==0 → *
```

2.9. Environmentally sensitive L-systems and Open L-systems

To model the impact of the environment on plants, and the mutual interaction between plants and their environment, two extensions to L-systems were made: environmentally sensitive L-systems and Open L-systems.

Environmentally sensitive L-systems [Pru1994] make it possible to pass information from the environment to the model. *Query modules* make it possible to access geometric information about the location and orientation of organs in the model. Query modules are produced in the axiom or in the production successors. After each derivation step the actual parameters of all query modules are set and then are used in the next derivation steps.

There are four main query modules: `?P(x, y, z)`, `?H(x, y, z)`, `?L(x, y, z)` and `?U(x, y, z)`. They correspond to geometric properties of the LOGO-style turtle: position, heading vector, left vector and up vector. When a query module is produced its parameters are set to arbitrary values. After each derivation step the string is interpreted (without drawing) and if a query module is found its actual parameters are set to the values corresponding to the current properties of the turtle. This phase is called *interpretation for the environment*.

Let us consider the following L-system:

```
Axiom: A
p1: A → F ?P(0, 0, 0) F ?P(0, 0, 0)
p2: F > ?P(x,y,z) : 1
     { printf("Line ends at (%f,%f,%f)\n", x, y, z); } → F
```

Initially the string contains a single module A. During the first derivation step the contents of the string is replaced with the sequence of four modules:

```
F ?P(0, 0, 0) F ?P(0, 0, 0).
```

The query modules `?P` have parameters set to 0 as specified in p_1. Then the interpretation for the environment follows. Let's assume that the turtle's initial position is (0,0,0) and the heading vector is (0,1,0). The first module found in the string during the interpretation is `F`. This module causes the turtle to move forward in the direction specified by its heading vector (see 2.3), so its position becomes (0,1,0). The next module found in the string is `?P`. Its three parameters will now be set to the values that represent the turtle's current position. So the contents of the string is *modified* and contains:

```
F ?P(0, 1, 0) F ?P(0, 0, 0).
```

Then the third module in the string is interpreted (`F`). It causes the turtle to move forward again. Now the turtle's position is (0,2,0). So when the next module is found (`?P`) its parameters are changed to contain (0,2,0). After the interpretation for the environment the string contains:

```
F ?P(0, 1, 0) F ?P(0, 2, 0).
```

This ends the first derivation step. During the interpretation for the environment the contents of the string was changed and the information from the environment acquired. In the second derivation step the production p_2 is applied twice. It doesn't change the string, but it prints two messages:

```
Line ends at (0,1,0)
Line ends at (0,2,0)
```

To illustrate the use of an environmentally-sensitive L-system a model[4] of an extinct plant *Horneophyton ligneri* is presented in Listing 11. The main feature captured in the model is the visible preference of the plant's branches to grow upwards rather than horizontally, which results in the characteristic shape of the plant's crown.

Listing 11 L-system generating a model of *Horneopython ligneri*

```
#define SENS 1.0 /* sensitivity to orientation */

derivation length: 7
Axiom: A(10)?H(0,0,0)

A(l)> ?H(x,y,z) → F(l)T
     [-(20)/(90)A(l*0.95*y^SENS)?H(0,0,0)]
     [+(20)/(90)A(l*0.95*y^SENS)?H(0,0,0)]

T → *

homomorphism

T → [-(20)/(90)F(3)][+(20)/(90)F(3)]
```

[4] Unpublished model by P. Prusinkiewicz

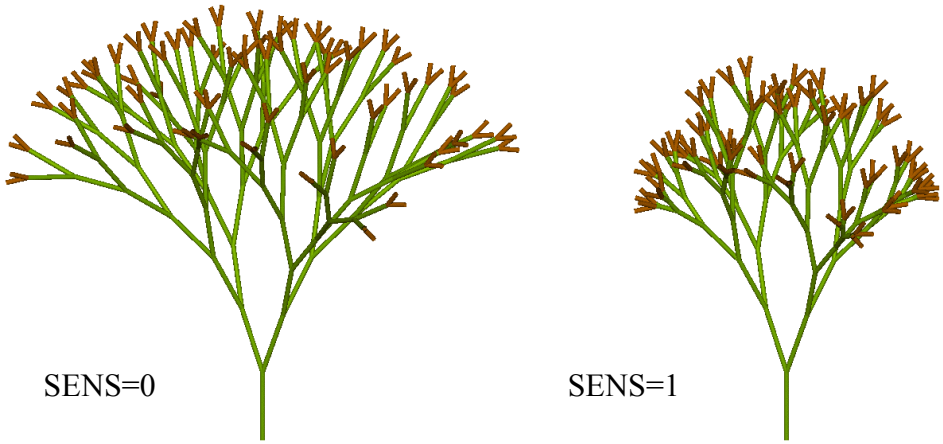


Figure 20 Image generated by the L-system from Listing 11 for two values of SENS

The presented model proposes a simple mechanism to capture this preference. It assumes that the length of branches produced by an apex depend on the vertical component of the apex's heading vector. It is possible to test the sensitivity of apices to the heading vector by manipulating parameter SENS that can accept any real-number values. The length of new branches is multiplied by the expression y^{SENS}, where `y` is the vertical component of the apex orientation vector. If, for example, SENS is equal to 0 (no sensitivity to the orientation), the generated structure presents no directional preference (see Figure 20 left). When SENS is set to 1 (Figure 20 right) the branches that grow more horizontally are visibly shorter than the ones growing more vertically.

To obtain the orientation vector of the apex, the query module `?H` is used. This module provides the model with the three components of the heading vector.

Where environmentally sensitive L-systems provide one-way communication from the environment to the plant model, Open L-systems [Mec1996] make it possible to model bi-directional interaction between plant and its environment. In this case, the task of modeling the environment is entrusted to an external program (usually written in a general purpose programming language such as C). The conceptual model behind open L-systems is presented in Figure 21.

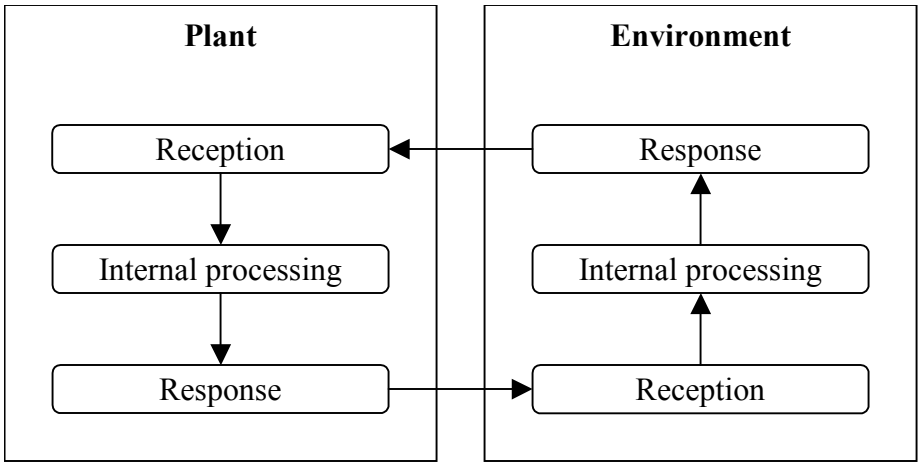


Figure 21 Conceptual model of interaction between plant and environment (after [Mec1997])

The *internal processing* phase in the plant model corresponds to a derivation step (cf. Figure 21). After each derivation step the string is scanned and communication modules together with optional additional information (e.g. position and orientation of the module in 3D) are sent to environment. The environment receives this information (*reception*), processes the data, and sends its *response* to the plant model. The plant model receives the response and is ready for the next derivation step. This feedback loop is continued throughout the simulation.

The exchange of information between the plant and its environment is done using *communication modules* `?E`. These modules are similar to the query symbols introduced in environmentally sensitive L-systems. The main difference is that when communication modules are generated their actual parameters are the input for the environment. This information is passed to the environment, which determines its response and sends back new values of parameters of the communication modules. These new values are then used in the productions.

To demonstrate how Open L-systems work, I am presenting a real-life example that demonstrates a phenomenon of canalization [Wad1942]: some organs (petals, primordia) in capitula are more likely to occur in certain quantities than in others. The model[5] presented in Listing 12 generates a planar, spiral phyllotactic pattern using the algorithm proposed by

[5] Unpublished model by P. Prusinkiewicz

Vogel in [Vog1979]. The algorithm places consecutive elements of the pattern (the primordia) using the following formulas:

$$\varphi = n * \alpha \qquad r = c\sqrt{n}$$

These formulas give the coordinates of pattern elements in the polar coordinates (r,φ). n is the ordering number and c is a scaling factor. Numbering starts at the centre and proceeds outward. Battjes and Prusinkiewicz [Bat1995] noticed that when generating phyllotactic pattern that contains N primordia using the Vogel formula the number of outermost primordia is usually a number from the Fibonacci series[6].

The L-system in Listing 12 generates phyllotactic patterns and demonstrates the effect of canalization of the number of organs.

Listing 12 Phyllotactic pattern and canalization of number of ray florets

```
Axiom: A(0)

C?E(x): x==0 --> @c
C?E(x): x==1 --> ;@c

decomposition

A(n) : n < NUMBER -->
  [+(n*137.5)f(0.5*n^0.5)C?E(n)]A(n+1)
```

[6] Fibonacci series is defined as follows: a_1=1, a_2 =1, a_n=a_{n-1}+a_{n-2} for n>2. The beginning of the series is: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55…

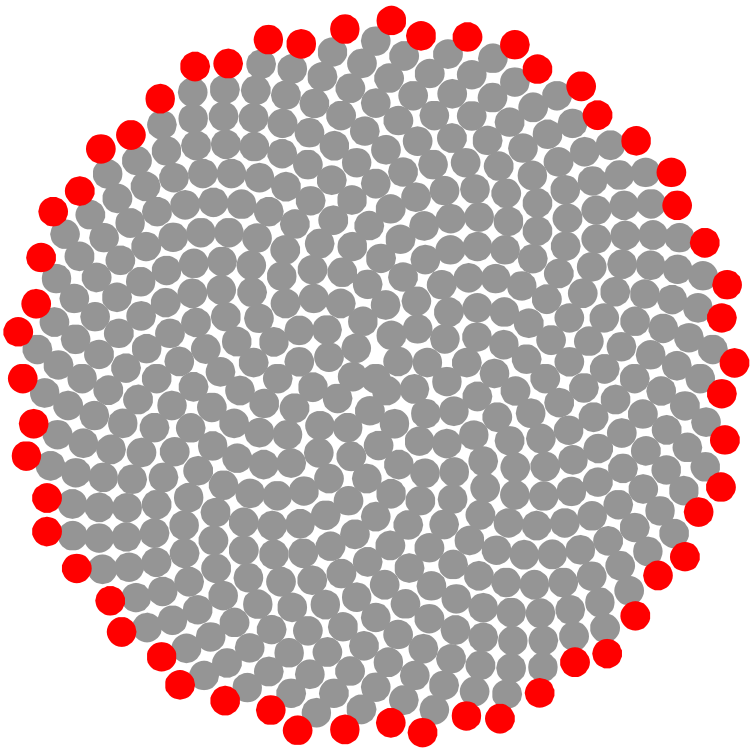


Figure 22 Phyllotactic pattern as generated by the L-system from Listing 12

The whole pattern is generated at once in the decomposition rule. Primordia are represented by modules `C` followed by a communication module `?E`. Each consecutive primordium has a parameter defining its vigour (`n`). The vigour is increasing as we move outward.

To determine which primordia are outermost, an environmental program is used. There are two pieces of information sent with every communication module `?E`: the position of the primordium (sent implicitly) and its vigour (`n`). The environment collects this information and determines which organs are *dominant*. A dominant organ is one that collides (occupies the same location in space as another organ) and has vigour that is *greater* than the organ with which it collides. The environment sends this information back by setting the value of the communication module `?E` parameter to 0 if the organ is dominated or to 1 if it is dominant. The dominating primordia are rendered using a different colour (red).

The simulation presented in Listing 12 was executed for the value of NUMBER in the range from 2 to 500. The results are shown on the chart on Figure 23. It can be seen that

most of the time the number of dominating primordia is a number from the Fibonacci series. These values are marked on the chart with the thick horizontal lines.

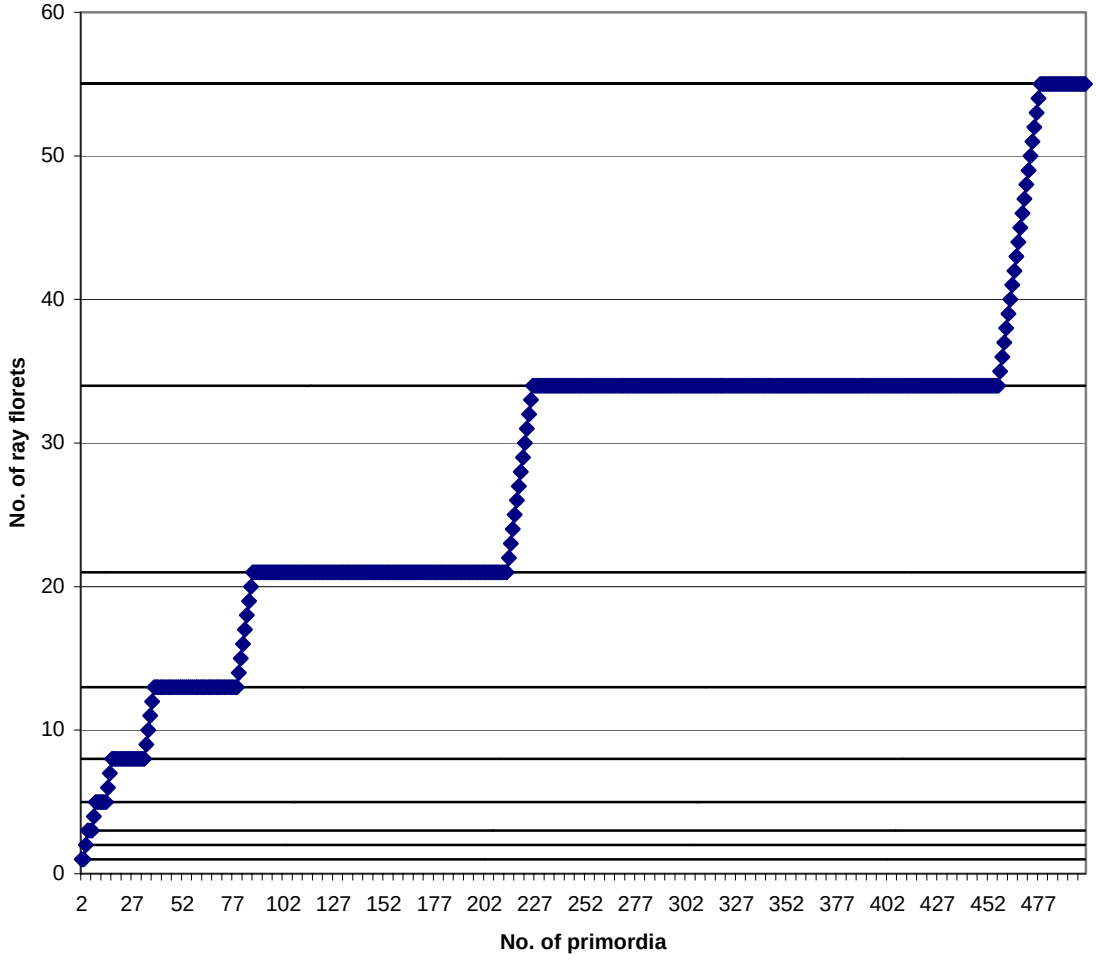


Figure 23 Results of the simulation from Listing 12

2.10. Summary

This chapter presented an overview of L-systems. It presents the main concepts that were incorporated into L-systems and turned a mathematical formalism into a powerful plant modeling language. All the concepts and the features presented in this chapter have been implemented in a plant modeling program *cpfg* [Mec1998, Pru1999]. This overview is intended to prepare the reader for the next chapter, where I present new concepts and features I have added to L-systems.

3. New Concepts and Features in L-systems

This chapter presents new concepts and features I have added to L-systems. Sections 3.1 and 3.2 describe why it is useful to extend parametric L-systems with user-defined data types, and user-definable functions, and how these extensions can be incorporated into the L-system formalism. In section 3.3 I present further extensions: control of derivation direction, fast information transfer and a modification of the notion of context: the new context.

3.1. User-defined data types

The introduction of parametric L-systems [Pru1990, Han1992] made it possible to include into the models quantitative information that can be expressed by real numbers. Examples of such information are:

- Geometric properties (length of an internode, diameter of a branch),
- Biologically relevant information (concentration of hormones, amount of nutrients produced/consumed),
- Biomechanical information (forces, torques, deformations),

For example, let us consider the production:

Listing 13 A sample parametric production

```
A(x,y) → B(x+y) C(x-y)
```

In this production a module of type `A` with two numerical parameters (`x` and `y`) is replaced by two modules, `B` and `C`, which have one parameter each. The values of the parameters associated with the newly created modules `B` and `C` are expressed using standard arithmetic notation. Specifically, the value of the parameter of module `B` equals the sum of parameters `x` and `y` of module `A`, and the parameter of `C` equals the difference, `x-y`.

Parametric L-systems made it possible to implement new classes of models. A module can have any number of parameters. However, n practice, if the number of parameters

associated with a module becomes too large it is very difficult to develop and maintain the model. For example:

Listing 14 A sample production with many parameters

```
A(xl1, xl2, xl3, xl4, xl5)
< B(x1, x2, x3, x4, x5) >
C(xr1, xr2, xr3, xr4, xr5)
→ D(x1, x2, x3, x4, x5+1)
```

Such a complex production is difficult to read and comprehend, although the concept is fairly straightforward: if a module `B` with five parameters has module `A` with five parameters as its left context and module `C` with five parameters as its right context, it is to be replaced with module `D` with five parameters. The values of the `D`'s parameters should be the same as those of `B`'s except that the last one that should be increased by 1.

According to the definition of parametric L-systems it is necessary to list all the formal parameters of each module involved in a production: both in the successor and in the predecessor. There are two main reasons for this:

- parameter passing is based on the position of the parameter, not its name,
- the modules are distinguished by the number of parameters, e.g. two modules labelled with the same letter but with a different number of parameters are considered to be two different modules. This is similar to the concept of overloaded functions [Str1991].

Although sometimes convenient, this formulation may also lead to bugs that are very difficult to find and fix. For example, it would be very easy to miss one of the parameters in a complex production such as Listing 15:

Listing 15 Production from Listing 14 with only four parameters in module D

```
A(xl1, xl2, xl3, xl4, xl5)
< B(x1, x2, x3, x4, x5) >
C(xr1, xr2, xr3, xr4, xr5)
→ D(x1, x2, x3, x5+1)
```

From the syntactical point of view the production would still be correct, therefore the parser would have no reason to indicate an error. But as a result every time the production in Listing 15 is applied, for example, it will produce a module `D` with *four* instead of *five* parameters. Consequently all the productions that have a module `D` with five parameters in their predecessors (whether in the strict predecessor or in the context) will never apply.

The solution proposed in this research is to allow the user to define compound data types, in particular structures. Using the syntax known from the C++ programming language the data type associated with modules in Listing 14 can be written as:

```
struct Data
{ float x1, x2, x3, x4, x5 };
```

The Listing 14 can then be expressed as follows:

Listing 16 Production from Listing 14 with parameters packed into structures

```
A(dl) < B(d) > C(dr) : { d.x5 += 1; } → D(d)
```

This production specification is significantly clearer than the previous one. It is important to notice that the type of parameters is not specified in productions. Looking at Listing 16 there is no way to say that `dl`, `d` and `dr` are of type `Data`. To avoid this kind of ambiguity the modules have to be declared before they can be used in productions (see 4.2).

3.2. Functions

As models developed using L-systems require more and more complex calculations the need for user-defined functions becomes apparent. In general the same reasons apply as in any other programming language.

- Functions encapsulate calculations. When a function is defined the calculations are separated from the rest of the program. At the same time the complexity of the calculations does not obstruct the main code.
- Functions are reusable: some formulas (or algorithms) are used in more than one place in a model. A simple call to the function can be used rather than repeating the details of the algorithm each time.

  Well-designed functions have meaningful names and good parameters that can give information about what the function actually does and what is the meaning of the parameters. When an algorithm is included in the main code it is necessary to look at the *implementation* to find out what it does. By using a function, a person reading the model can understand *what* the algorithm does, without having to know *how* it does it.
- Functions can be much more expressive than arithmetic formulas. Some languages that do not support functions include simple macros (in the C preprocessor sense) for representing calculations[7]. For example:

  ```
  #define VectorLength(x, y, z) sqrt(x*x+y*y+z*z)
  ```

  However no loops or conditional statements can be used in macros.
- Functions can be combined into libraries: Some functions are used in different models. For example functions that calculate the dot product of vectors and the length of vectors are general in nature and are used in different models. Instead of rewriting them one can group such functions into a library that can be linked to different models.

The calculations of L-systems do not require any special syntax. It is therefore reasonable to assume that the syntax from any general-purpose language is acceptable.

3.3. Fast information transfer

The importance of information transfer in biological models cannot be overstated. For example, information transfer (or signal propagation) in L-system models is used to model

[7] C preprocessor is used for example by *rayshade*. A mechanism similar to C preprocessor is also used in POV-ray.

the transport of substances in the plant, such as hormones, nutrients. It is also used for collecting information about the structure, such as number of apices supported by a given branch, and biomechanical information such as the: sum of forces, momentums, and torques acting on a branch. Information can be transferred using context-sensitive productions but this mechanism may not be fast enough in some applications. In the following sections I present the traditional approach to the problem and then introduce a new method, which I call *fast information transfer*.

3.3.1. Information transfer in linear structures

This section discusses the problem of information transfer in linear structures. First the traditional approach using context-sensitive productions is presented in 3.3.1.1. Then the new method using fast information transfer is shown in 3.3.1.2.

3.3.1.1. Traditional approach

Context-sensitive L-systems as introduced by Lindenmayer [Lin1968] have been a natural choice for expressing information transfer. For example the L-system in Listing 17 moves a signal from left to right through the series of `A` modules, counting the number of `A`'s.

Listing 17 Information transfer in linear structure using context-sensitive productions

```
axiom: S(0) AAAAAAAA
S(n) < A → A S(n+1)
S(n) > A → ε
S(n) → R(n)
```

`S` has one parameter, which represents the number of `A`'s counted so far. In the axiom this value is set to 0. After the first derivation step module `S` will move past the leftmost `A` and the value of its parameter will be 1:

```
AS(1) AAAAAAA
```

After the next derivation step `S` will move past the second `A` and the value of its parameter will increment:

```
AAS(2) AAAAAA
```

Eventually after eight steps module `S` will reach the end and will be replaced with module `R` representing the final result.

```
AAAAAAAA R(8)
```

It takes *N* derivation steps to transfer the signal (the counter module `S`) from one end of the string to the other, where *N* is the number of `A`'s. Because the time required to perform a single derivation step is also proportional to *N*, this method of transferring information is $O(N^2)$.

3.3.1.2. Fast information transfer

According to the original formulation of L-systems the modules in the string are replaced with their successors *simultaneously*. Nevertheless, in practice the process of derivation is usually performed sequentially. The original string is scanned, module by module, and as the productions are applied the new string is built. If the user *knows* that the process of derivation is performed in a given direction (for example from left to right) it is possible to rewrite Listing 17 as follows:

Listing 18 Fast information transfer in a linear structure, using a global variable

```
n = 0;
axiom: AAAAAAAA
p1: A > A : { n++; } → A
p2: A : { n++; } → A R(n)
```

In a single derivation step, the first production replaces each module `A` that has another module `A` to the right with itself, while the second production handles the end case. Both productions increment `n` as each `A` is replaced. The table below shows the execution of a single derivation step. The first column shows the contents of the original string and the current module is shown in boldface.

Original string	Production applied	n	Resulting string
AAAAAAAA	p_1	1	A
A**A**AAAAAA	p_1	2	AA
AA**A**AAAAA	p_1	3	AAA
AAA**A**AAAA	p_1	4	AAAA
AAAA**A**AAA	p_1	5	AAAAA
AAAAA**A**AA	p_1	6	AAAAAA
AAAAAA**A**A	p_1	7	AAAAAAA
AAAAAAA**A**	p_2	8	AAAAAAAA R(8)

In this way the signal propagates through the string in a single derivation step and the string is only scanned once. Since the time required to perform a derivation step is proportional to the number of modules in the string, this method is $O(N)$.

If a signal needs to be propagated from right to left, the analogous method can be applied as long as the derivation can be performed from right to left. So here I am introducing the term *derivation direction*. I assume that the process of string rewriting is done sequentially. The derivation direction specifies whether during rewriting the string is scanned from left to right (*forward*) or from right to left (*backward*).

The information propagation presented in this section is an example of using the fast information transfer in a linear structure. The only overhead in this method compared to the traditional method is the need for a global variable (n). To use the fast information transfer method, the derivation direction must be controlled by the user.

3.3.2. Information transfer in branching structures

Examples from the previous sections can be extended to branching structures where it is often necessary to count the branching order to determine features such as the age of a branch or its distance from the base. Berntson [Ber1997] presented a number of topological ordering schemes, which can be divided into two broad categories: (*developmental*) *centrifugal* and (*functional*) *centripetal*.

When labelling segments according to a developmental scheme, the process starts at the base of the system and orders are assigned in increasing magnitude away from the base. The name *developmental* reflects the direction of growth of the branching (or root) system.

When a *functional* scheme is used, the process starts at the tips of the branches (or root tips) and orders of increasing magnitude are assigned toward the base. This scheme reflects the distance (especially topological distance) to the base and is usually correlated with age of the organ.

Two different labelling schemes will be applied on simple branching structures. One of the schemes requires a signal to be propagated from tips of branches to the root and the other requires the signal to go from the base of the structure to the tips. Both labeling schemes will be first implemented using context-sensitive L-systems and then using fast information transfer.

Here is a sample branching structure:

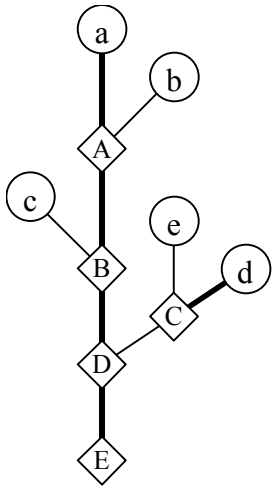


Figure 24 A sample branching structure

The structure in the Figure 24 corresponds to the following string:

$$\texttt{I[I[IA}_{\texttt{e}}\texttt{]IA}_{\texttt{d}}\texttt{]I[IA}_{\texttt{c}}\texttt{]I[IA}_{\texttt{b}}\texttt{]IA}_{\texttt{a}}$$

Modules `I` represent internodes (lines). Modules `A` represent apices (circles). The indices associated with modules of type `A` have been added only to help identify the modules on the diagram and those in the string. Branching points are visualized as diamonds and are labelled with capital letters. Internodes that are lateral to their parent branch are thinner than main branches. For example the internode between the branching points `D` and `B` is a main branch, whereas the internode from `B` to the apex `c` is lateral. On the other hand internode `D C` is lateral to its parent (`E D`), but the internode from `C` to apex `d` is the main

branch for `D C`. The distinction between the main and lateral branches will become relevant when implementing developmental labelling scheme in the section 3.3.2.2.

3.3.2.1. Traditional approach

Developmental labelling scheme

In the case of developmental labelling schemes the signal needs to be propagated from the root to the tips. The root as the oldest part of the plant has number 1. All child branches (main or lateral) have numbers based on the label of their parent incremented by one.

The L-system that performs the labelling contains two productions:

Axiom: $I[I[IA_e]IA_d]I[IA_c]I[IA_b]IA_a$

p_1: I(parent)< I(n) → I(parent+1)

p_2: I(n) → I(1)

And this is how it proceeds:

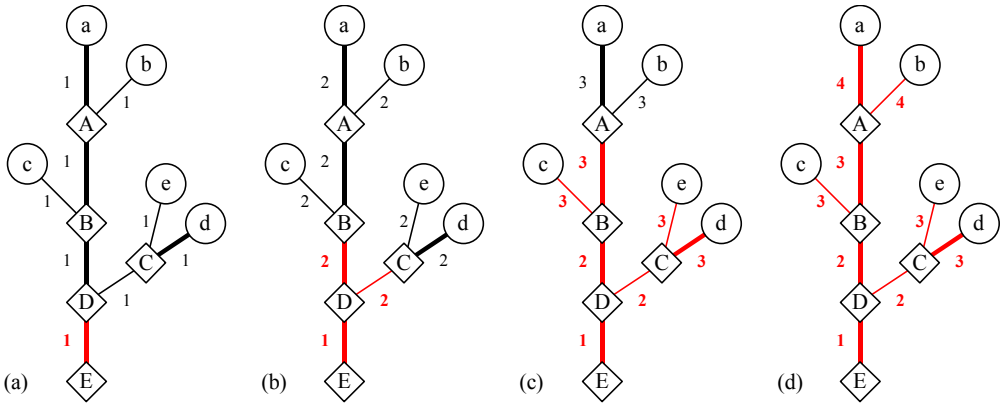


Figure 25 Information transfer in a branching structure from the root to the tips

Red lines represent the internodes, which contain updated information. The number of steps required to propagate the information is equal to the length of the longest branch.

Functional labelling scheme

This labelling scheme can be used to calculate the number of apices supported directly or indirectly by every internode. Every module representing an internode `I` will have a

parameter for storing this information. Using context sensitive L-systems the solution can be described as follows:

1) The initial value of the internode's parameter is equal to 0 for all modules `I`.
2) Where internodes directly support an apex the value of the parameter is set to 1.
3) For all other internodes the parameter is calculated by adding the parameters from the internodes to their right

p_1: `I(n) > A → I(1)`
p_2: `I(n) > [I(n1)] I(n2) → I(n1+n2)`

If the structure contains any sequences of two consecutive internodes without a branching point an additional production is required:

p_3: `I(n) > I(n1) → I(n1)`

As the productions are applied in each derivation step the information about the number of apices supported propagates downwards until it reaches the base:

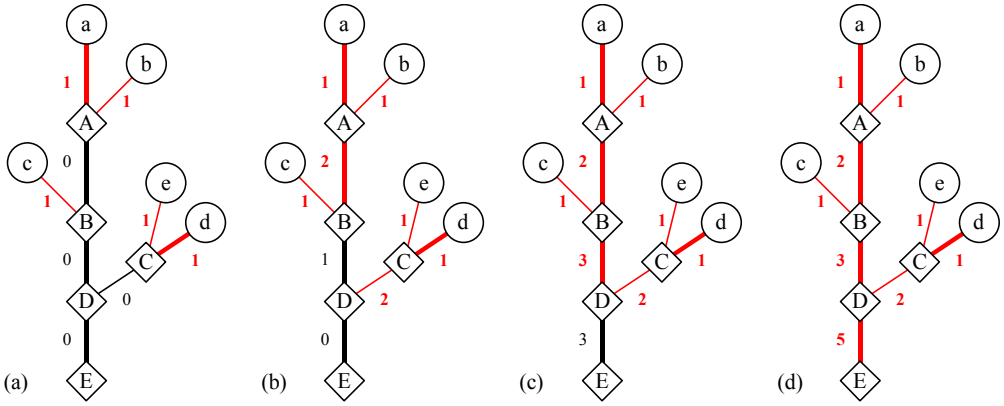


Figure 26 Information transfer in a branching structure from the tips to the root

Figure 26 shows how the information propagates. The red lines indicate internodes to which the information has been propagated. The numbers indicate the actual values of parameters of `I`.

In the case of asymmetric branching structures (as the one presented in this example) the information may not come simultaneously to branching points. For example in Figure 26

(a) the information has reached point `B` only from `c` but not from `A`. In (b) the information has reached the point `B` but `D` is still waiting, etc.

It is imperative to be able to determine whether information has reached a certain point and yet this is a non-trivial task. For example, checking if the value is non-zero can be misleading (e.g. the parameter of internode `DB` in figure (b) or the parameter of internode `ED` in figure (c) are non-zero but do not contain the correct values). In practice it is necessary to use an additional parameter (flag) to indicate if the information has reached the given point.

The number of steps required to propagate a signal in a branching structure is equal to the length of the longest branch (main or lateral).

3.3.2.2. Fast information transfer with a stack

In the case of a branching structure, a single variable is usually not enough to apply fast information transfer. Instead a stack is required. It is assumed that there is a data structure `theStack`, which implements a stack of integers. `theStack` implements three methods: `void Push(int)` and `int Pop()` (removes the top element from the stack) and `int Top()` (returns the value of the top element without removing it).

Developmental labelling scheme

To propagate a developmental signal in a branching structure it is necessary to be able to distinguish between main and lateral branches. The derivation must be performed forward. In the following L-system the second parameter of `I` indicates main and lateral internodes. It is `true` (1) for the main internodes and `false` (0) for the lateral ones:

Listing 19 Fast information transfer applied to a developmental signal in a branching structure

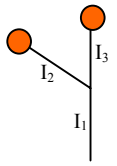
```
p1: I(nl, sl) < I(n,s) :
{
  if (s==true)
    newn = theStack.Pop()+1;
  else
    newn = theStack.Top()+1;
  theStack.Push(newn);
} → I(newn, s)

p2: I(n,s) < A() :
{ theStack.Pop(); } → A
```

p_3: I(n,s) :
{ theStack.Push(1); } → I(1,s)

When a new value of I's first parameter is set (p_1 and p_3) this value is pushed on the stack. When the value of the previous label is needed (p_1) it is accessible as the top element on the stack.

This value is only read when the current internode is a lateral branch and is read and removed from the stack when the current internode is a main branch. This is because every lateral branch precedes a main branch in the string. To analyze how this L-system works a simpler branching structure was chosen (see Figure 27).



$I_1[I_2A]I_3A$

Figure 27 Sample branching structure

The table below shows the progress of a single derivation step performed on the string representing the branching structure (Figure 27). In the first column the current module is indicated in boldface. The third column shows the contents of the stack after the production specified in the second column has been applied.

Original string	Production applied	Stack	Resulting string
$\mathbf{I_1(0,1)}[I_2(0,0)A]I_3(0,1)A$	p_3	1	$I_1(1,1)$
$I_1(0,1)[\mathbf{I_2(0,0)}A]I_3(0,1)A$	p_1	2 1	$I_1(1,1)[I_2(2,0)$
$I_1(0,1)[I_2(0,0)\mathbf{A}]I_3(0,1)A$	p_2	1	$I_1(1,1)[I_2(2,0)A$
$I_1(0,1)[I_2(0,0)A]\mathbf{I_3(0,1)}A$	p_1	2	$I_1(1,1)[I_2(2,0)A]I_3(2,1)$
$I_1(0,1)[I_2(0,0)A]I_3(0,1)\mathbf{A}$	p_2	*Empty*	$I_1(1,1)[I_2(2,0)A]I_3(2,1)A$

Every internode (except the first one) uses the information from the stack to calculate its own order. Also productions push the order value on the stack to be used by the next internode. Terminal internode values are not used; therefore they must be removed from the stack (p_2). Also every production for a main internode removes the top element from the stack, as it is no longer used.

Functional labelling scheme

This L-system implements the functional scheme presented in 3.3.1.2 using fast information transfer.

Listing 20 Fast information transfer in a branching structure using a global stack

```
p1: I(n) > A : { theStack.Push(1); } → I(1)

p2: I(n) > [I(nl)] I(ns) :
{
  n = theStack.Pop() + theStack.Pop();
  theStack.Push(n);
} → I(n)
```

This L-system assumes that the derivation is performed backwards – from right to left. This is how the process proceeds on the branching structure presented in Figure 27.

Original string	Production applied	Stack	Resulting string
$I_1(0)[I_2(0)A]\mathbf{I_3(0)}A$	p_1	1	$I_3(1)A$
$I_1(0)[\mathbf{I_2(0)}A]I_3(0)A$	p_1	1 1	$I_2(1)A]I_3(1)A$
$\mathbf{I_1(0)}[I_2(0)A]I_3(0)A$	p_2	2	$I_1(2)[I_2(1)A]I_3(1)A$

The algorithm is based on the fact that the branch tips are visited before the branching points. When a branching point is visited, the stack contains the values pushed by the productions applied for the internodes at the tips. Every internode pushes the value of its parameter on the stack. Internodes at the tips (those that have an apex in the right context) set the value of the parameter to 1. Internodes at the branching points calculate the value of the parameter based on two topmost values on the stack. This method resembles the way arithmetic expressions are calculated using the reverse Polish notation.

3.3.2.3. Fast information transfer using the new context

The method presented in the previous sections requires a global data structure – the stack. The stack contains data that are a subset of the information that is generated in successors. But in fact these data are redundant as they are already present in the new string being created.

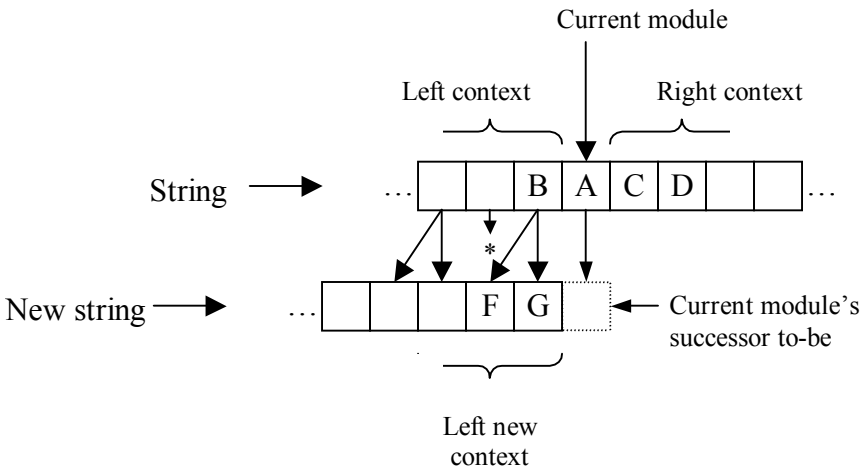


Figure 28 Left context, right context and new left context

In Figure 28 the current module is A. Modules in the immediate neighbourhood to the left of module A are its left context. Modules in the immediate neighbourhood to the right of module A are its right context.

Here I introduce a new concept: *new context*.

To determine if a module or modules are in left new context of module A the same rules for matching apply as described in 2.5.1. The only difference is that matching is performed in the new string (the one being currently created) starting at the last module added to the new string so far. Left new context is defined only when the derivation is being performed forward (from left to right).

Similarly right new context is defined only when deriving from right to left. Also the rules described in 2.5.1 apply. In Figure 29 the string is being derived backwards (from right to left). Modules FG etc. are the right new context of the current module A.

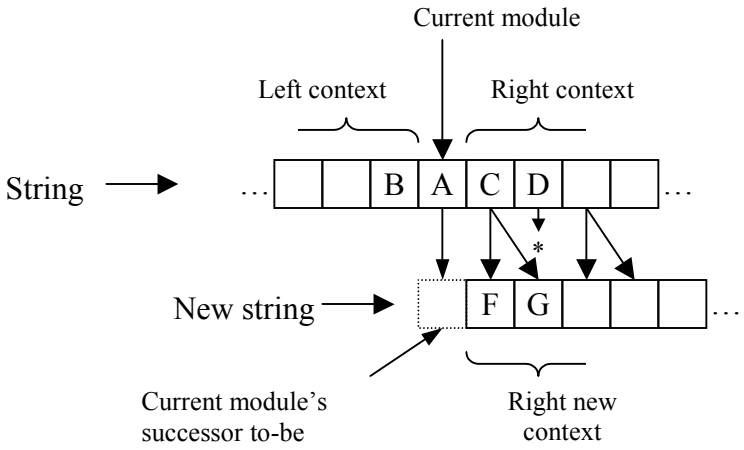


Figure 29 Left context, right context and new right context

The stack as it is used in the fast information transfer can be thought of as a method of indirectly reading the contents of the string being generated. But once the formalism makes it possible to explicitly refer to the newly created modules, L-systems that implement fast information transfer are simpler and there is no need for the global stack. I am introducing new meta-symbols: << and >>. These symbols specify the new left and new right context, respectively. The introduction of these meta-symbols adds four new types of production predecessors:

left new context <<	*strict predecessor*	
left new context <<	*strict predecessor*	*> right context*
	strict predecessor	*>> new right context*
left context <	*strict predecessor*	*>> new right context*

It makes sense to match productions with left new context only when the string is being derived forward (from left to right). Similarly, productions that have new right context can be matched only when deriving the string backwards (from right to left). Obviously it makes no sense to have a production that would have both left and right new contexts as the string cannot be derived in both directions at the same time. We are now ready to re-implement the two labelling schemes (Listing 19 and Listing 20) without using the global stack.

Developmental labelling scheme

An L-system using the fast information transfer with new context that implements the developmental labelling scheme contains two productions is presented below:

Listing 21 Developmental labelling scheme implemented using fast information transfer with new context

```
p1: I(nl) << I(n) → I(nl+1)
p2: I(n) → I(n+1)
```

The process of rewriting is shown in the table below:

Original string	Production applied	Resulting string
I(0)[I(0)A]I(0)A	p_2	I(1)
I(0)[**I(0)**A]I(0)A	p_1	I(1)[I(2)
I(0)[I(0)**A**]I(0)A	*identity*	I(1)[I(2)A
I(0)[I(0)A]**I(0)**A	p_1	I(1)[I(2)A]I(2)
I(0)[I(0)A]I(0)**A**	*identity*	I(1)[I(2)A]I(2)A

The L-system using the new context has the following advantages:

- There is no need for the stack
- It needs just one production (production p_2 merely initializes the first internode's parameter to 1)
- There is no need for an extra parameter to distinguish between main and lateral branches

Functional labelling scheme

The L-system in Listing 22 implements the functional labelling scheme using the new context. The derivation is performed backwards.

Listing 22 Functional labelling scheme implemented using the new context

```
p1: I(n) > A → I(1)
p2: I(n) >> [ I(nl) ] I(ns) → I(nl+ns)
```

The table below shows the process of rewriting

Original string	Production applied	Resulting string
I(0)[I(0)A]I(0)**A**	*identity*	A
I(0)[I(0)A]**I(0)**A	p_1	I(1)A
I(0)[I(0)**A**]I(0)A	*identity*	A]I(1)A
I(0)[**I(0)**A]I(0)A	p_1	I(1)A]I(1)A
I(0)[I(0)A]I(0)A	p_2	I(2)[I(1)A]I(1)A

3.4. Summary

This chapter presents the extensions I have added to the formalism of L-systems. The extensions include two features known from general purpose languages: user-defined data types and functions. These features extend the capabilities of L-systems in expressing models that require many parameters and complex calculations.

A new concept, not found in other formalisms is the concept of fast information transfer. Fast information transfer together derivation direction and new context make it possible to transfer signals through the structure represented by the L-system string in a single derivation step.

All these extensions and concepts have been incorporated in the modeling language L+C, which is described in the following chapter.

4. The Modeling language L+C

The number and significance of the new features and concepts that I have added to the formalism of L-systems made it desirable to design a new language instead of extending the existing implementation of the plant modeling language *cpfg*. This is why I have designed modeling language L+C[8]. This language is a declarative language based on the formalism of L-systems.

L+C combines constructs, which can be divided into two categories:

a) constructs specific to L+C
b) constructs known from other programming languages, in particular the C++ general purpose programming language.

The syntax of the constructs that are not characteristic to L-systems has been borrowed from C++. This includes rules for scoping. Contents specific to L-systems have syntax, which is partially inherited from the traditional notation of L-systems, but also some effort has been made to use syntax that would not look too alien to C++. The decision of designing L+C based on the syntax of C++ gives the following benefits:

- the learning curve is gentle for people who already know C++,
- the expressive power of C++ together with existing methodologies and libraries can be used directly in an L+C program,
- no documentation is needed for the C++ part of the language.

A program in L+C consists of a series of declarations:

- Structures, classes
- Global variables
- Functions
- Derivation length
- Modules

[8] First draft of the language specifications, has been prepared together with P. Prusinkiewicz, R. Sievänen and J. Perttunen and described in an unpublished manuscript.

- Axiom
- Productions
- Decomposition rules
- Interpretation rules
- Control statements

This chapter discusses the syntax of L+C. As the syntax of C++ constructs is the same as in C++, only the syntax of constructs specific to L+C will be presented. Listings in this chapter show sample uses of the constructs being presented.

4.1. Derivation length

Derivation length specifies the number of derivation steps.

`derivation length:` *expression*`;`

4.2. Module declarations

L+C requires that all modules used in a model must be declared. Modules are identified by their names (identifiers). Two modules are predefined in L+C: `SB` and `EB`. `SB` is the start of a branch and `EB` is the end of a branch. These modules correspond to the modules `[` and `]` in the traditional notation.

Module declaration has the following syntax:

`module` *identifier*`(`*parameter-list*$_{opt}$`);`

The identifier must be a valid C++ identifier. The parameter list is an optional list of type identifiers separated by commas. The following listing presents some sample module declarations:

Listing 23 Examples of module declarations

```
struct data
{
  int n;
  float arr[3];
```

```
};

// module without parameters
module A();
// module with two parameters
module C(int, float);
// module with a parameter of user-defined structure type data
module Internode(data);
```

Types that are used as module parameter types must be defined beforehand.

4.3. Axiom

Axiom defines the initial contents of the L-system string. There must be exactly one axiom declared in every L-system. The syntax of the axiom is as follows:

`axiom:` *parametric-word*$_{opt}$`;`

A parametric word is a sequence of one or more parametric modules:

identifier`(`*expression*$_1$`,` *expression*$_2$`, ...)`

Where *identifier* must be name of a previously declared module. The number of expressions in the parenthesis must be the same as the number of parameters in the module's declarations. Also, the types of expressions must correspond to the types of parameters in the module's declaration. If the parameter is declared to have no parameters (as module A in Listing 23), then the module's identifier is followed by (). Optionally the parentheses might be skipped altogether.

If modules `A`, `C` and `Internode` are declared as in Listing 23 a sample axiom may look like this:

```
axiom: A C(i, 2.3) Internode(d);
```

Here, `i` must be an integer and `d` must be a variable of type `data`. The same syntax of parametric word is also used in the `produce` statement.

4.4. Productions

Productions define the way the structure represented by the L-system string develops over time by specifying the fate of every module. Declaration of a production starts with the predecessor. The predecessor consists of three components: the strict predecessor, the left context and the right context (see 2.5). The left context and the right context are optional. Also, one of the new contexts (left or right) can be specified in a production predecessor (see section 3.3.2.3). The strict predecessor specifies which module(s) in the string will be replaced.

Left and right context specify which modules must be present in the neighbourhood of the strict predecessor. The general syntax for productions is:

leftcontext$_{opt}$ < *newleftcontext*$_{opt}$ << *strictpred* >> *newrightcontext*$_{opt}$ > *rightcontext*$_{opt}$:
{
    production body
}

All components of a production predecessor are sequences of one or more modules with formal parameters, such as those shown in the following example:

Listing 24 Example of production predecessor in L+C

```
A() < C(i, r) > Internode(d)
```

This predecessor specifies that the production should be applied to module `C` if there is a module `A` to the left of `C` and a module `Internode` to the right. The types of parameters are determined by the declarations of modules, which have to appear before the modules are used in a production. If the modules `A`, `C` and `Internode` are declared as in Listing 23 then `i` will be an integer, `r` will be a float and `d` will be a structure of type `data`. All formal parameters of every module must be listed, even if they are not used in the production body. The identifiers of formal parameters must be unique within a production. Their scope is the same as the scope of formal parameters in C++ functions.

Formal parameters are similar to formal parameters of functions. When a production is applied they are given values. These values can be used inside the production body.

In the case of a production, the formal parameters get the values from the actual parameters of modules in the string. For example, if the string contains the following sequence:

```
A C(5, 2.5) I({ 2, { 0, 0.4, 0.6 }})
```

the production from Listing 24 can be applied. The production formal parameters will be assigned the following values:

```
i = 5, r = 2.5, d = { 2, { 0, 0.4, 0.6 }}
```

A program in L+C can define more than one production for the same module. In other words more than one production can refer to the same module in the strict predecessor.

Listing 25 Sample production predecessors in L+C

```
A() < C(i, r) > Internode(d) :
{ … }

C(i, r) > Internode(d) :
{ … }

C(i, r) :
{ ... }
```

When more than one production is declared for a given module, the order in which these productions are specified is important. When the string is being rewritten, then for every module in the string a production must be applied. If there is more than one production for a given module then the productions will be tested for matching *in the order in which they are declared*. In Listing 25 there are three productions for module C. The first one specifies both the left and right context, the second specifies only the right context and the third one is context-free.

According to the original definition [Lin1968] productions are applied in parallel. In L+C the derivation is performed sequentially[9]. It can be performed in one of two directions: forward (from left to right) or backward (from right to left) (see 3.3).

Because the derivation is performed in a specified direction and this direction can be controlled, L+C allows the use of the new context (see 3.3.2.3). The syntax used for specifying the new context is the same as that used with ordinary context. Checking for new and ordinary context can be mixed in one production, as in the following examples:

```
A() < C(i, r) >> Internode(d) : …
A() < A() << C(i, r) > Internode(d) : …
A() << C(i, r) : ...
A() < C(i, r) >> Internode(fd) > Internode(d) : ...
```

etc.

As explained in section 3.3.2.3 checking for right new context makes sense and is possible only when the string is being derived from right to left. If the derivation is being performed in the opposite direction, these productions are ignored. Similarly, checking for the left new context is possible only when the string is being derived from left to right and this is the only time when these productions are tested for matching. The derivation direction is controlled by the user form within L-system (see A.6.13). Discussion of the use of new context is presented in section 3.3.2.3.

A production successor is specified using the *produce statement*. The syntax of the produce statement is presented in the following section.

4.5. The produce statement

A production consists of the predecessor and the body. The production body is a compound statement that can contain any code allowed inside a C++ function. In addition,

[9] As a matter of fact, the introduction of the *cut* module [Han1992] silently implies that the process of derivation is performed sequentially from left to right.

productions specify successors. In L+C the successor of a production is specified in the produce statement.

The produce statement has the following syntax:

`produce` *parametric-word*$_{opt}$;

The parametric word has the same syntax as in the axiom (see 4.3). In addition to specifying the successor the produce statement also terminates the production.

In general production execution can be terminated in one of two ways:

1) by a produce statement,
2) by control flow leaving the scope of the production in any other way (such as end of code, return statement)

If a production is not terminated by the execution of a produce statement, the production is considered as *not applied* and another production will be searched for matching.

The syntax of L+C constructs presented so far allows one to write a simple model. The following program specifies the development of a branching structure:

Listing 26 L-system generating simple branching structure

```
#define dt 0.03
#define t_max 1.0

module A(float);
module I(float);

axiom: A(0);

derivation length: 100;

p1: A(t) :
{
  float new_t = t+dt;
  if (new_t<=t_max)
    produce A(new_t);
}

p2: A(t) :
{
  float new_t = t+dt;
  if (new_t>t_max)
  {
    float t_init = new_t - t_max;
    produce
```

```
        I(t_init) SB A(t_init) EB
        SB A(t_init) EB
        I(t_init) A(t_init);
    }
}
```

p_3: `I(t) : { produce I(t+dt); }`

The model implemented by this program is expressed using two types of modules: apices (module `A`) and internodes (module `I`). Each of these modules has one numerical parameter, which corresponds to the age of the organ. Initially the structure consists of a single apex of age 0. At every derivation step the age of apex is increased by `dt` (p_1). When an apex reaches mature age (p_2) it produces an internode, two lateral apices and another internode followed by an apex (see Figure 30). The last production specifies that internodes grow older at every time step by `dt`.

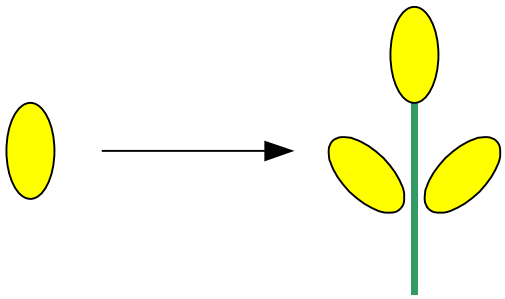


Figure 30 Apex producing internodes and new apices

4.5.1. Multiple successors

The code in Listing 26 includes a feature that deserves more attention. Two productions are defined for module `A` (p_1 and p_2). In (p_1) a local variable `new_t` is declared and its value calculated based on the value of `A`'s parameter t. If condition `(new_t<=_max)` evaluates to true then a `produce` statement is executed. Otherwise the control flow leaves the scope of the production. In this case the production is considered *not applied* which means that the next production (p_2) will be tested for matching. p_2 defines and initializes `new_t` as in the previous production and if condition `(new_t>t_max)` is met then the `produce` statement is executed.

In L+C it is possible to combine these two productions into one production:

Listing 27 Production with multiple successors

```
A(t) :
{
  float new_t = t+dt;
  if (new_t<=t_max)
    produce A(new_t);
  else // (new_t>t_max)
  {
    float t_init = new_t - t_max;
    produce
      I(t_init) SB A(t_init) EB
      SB A(t_init) EB
      I(t_init) A(t_init);
  }
}
```

Because every produce statement defines the successor this production has two successors. In general the ability to define multiple successors in a single production is a significant improvement in the expressiveness of L-systems.

4.5.2. Empty successor

To remove a module from the string the production must specify an empty successor. In L+C the keyword `produce` followed by a semicolon specifies empty successor.

4.6. Decomposition rules

Decomposition rules express the concept of compound modules (see 2.8). Decomposition rule

```
A() : { produce B C; }
```

states that module `A` *consists* of modules `B` and `C`. Decomposition rules can also be defined for modules B and C, and they will be applied recursively as long as there are modules that can be further decomposed.

In L+C decomposition rules are always context-free. Decomposition rules are preceded by the `decomposition:` keyword. The result of decomposition is stored in the L-system string permanently. The model below is the same as the one presented in Listing 26 but it uses a decomposition rule to divide an apex into internodes and new apices.

Listing 28 L-system based on Listing 26 using decomposition rules

```
#define dt 0.03
#define t_max 1.0

module A(float);
module I(float);

axiom: A(0);

derivation length: 100;

p1: A(t) : { produce A(t+dt); }

p2: I(t) : { produce I(t+dt); }

decomposition:

d1: A(t) :
{
  if (t>t_max)
  {
    float t_init = t - t_max;
    produce
      I(t_init) SB A(t_init) EB
      SB A(t_init) EB
      I(t_init) A(t_init);
  }
}
```

The decomposition rule (d_1) in Listing 28 contains a condition (`t>t_max`), which makes sure that apices are not decomposed infinitely by imposing a terminating condition. If a decomposition rule does not contain a terminating condition (or if there is a bug and the condition is never satisfied), an infinite recursion could result. To avoid infinite recursion a safeguard parameter called *maximum decomposition depth* can be specified using the following syntax:

`maximum depth:` *expression*`;`

If no maximum depth is specified then a default value is used. If the maximum depth is reached during the execution of a program then a run-time warning message will be printed.

4.7. Interpretation rules

Interpretation rules are used to separate the visual aspect of models from the developmental aspect (see 2.7). Modules generated by interpretation rules are interpreted directly by the graphics engine.

Below is the third version of the model presented in Listing 26, which includes geometrical information about the organs (apex and internode) as well as their graphical representation.

Listing 29 L-system based on Listing 26 with interpretation rules

```
#define dt 0.03
#define t_max 1.0
#define angle 35

module A(float);
module I(float);

axiom: A(0);

derivation length: 100;

A(t) : { produce A(t+dt); }

I(t) : { produce I(t+dt); }

decomposition:

A(t) :
{
  if (t>t_max)
  {
    float t_init = t - t_max;
    produce
      I(t_init) SB Left(angle) A(t_init) EB
      SB Right(angle) A(t_init) EB
      I(t_init) A(t_init);
  }
}

interpretation:

A(t) : { produce SetColor(1) Circle(0.2*t); }

I(t) : { produce SetColor(2) F(t); }
```

The graphical aspects of the model's representation include:

1) Lateral apices are orientated relative to the main branch (modules `Left` and `Right` in the decomposition rule)
2) Interpretation rules specify that apices should be visualized as circles and internodes as lines (module `F`). Apices and internodes are rendered using two different colours (modules `SetColor` in the interpretation rules).

4.8. Control statements

There are four control statements in L+C: `Start`, `StartEach`, `EndEach` and `End`. Control statements are procedures that are called at specific points of the execution of an L+C program: at the beginning, before every derivation step, after every derivation step and after the last step, respectively. The following sample program demonstrates how to use control statements to output results of a simulation to an external file. This program is another modification of the model presented in Listing 26:

Listing 30 L-system based on Listing 26 using control statements and file I/O

```
#define dt 0.03
#define t_max 1.0

module A(float);
module I(float);

axiom: A(0);

derivation length: 100;

int stepno, apexcount;
FILE* fpOutput;

Start:
{
  fpOutput = fopen("output.dat", "w");
  stepno = 0;
}

StartEach:
{
  apexcount = 0;
  stepno++;
}

EndEach:
{
  fprintf(fpOutput,
    "Step: %d\t, Number of apices: %d\n",
```

```
    stepno, apexcont);
}

End:
{
  fclose(fpOutput);
}

A(t) :
{
  float new_t = t+dt;
  if (t<=t_max)
  {
    apexcount++;
    produce A(new_t);
  }
  else // (t>t_max)
  {
    float t_init = t - t_max;
    apexcount += 3;
    produce
      I(t_init) SB A(t_init) EB
      SB A(t_init) EB
      I(t_init) A(t_init);
  }

}

I(t) : { produce I(t+dt); }
```

In addition to generating the branching structure this program also stores some statistical information in an external file (`output.dat`). In the `Start` statement it opens the external file `output.dat` and sets the `stepno` (step number) variable to 0. Before every derivation step `StartEach` is executed where `apexcount` (apex counter) is set to 0 and `stepno` is incremented. After every derivation step `EndEach` is executed which writes to the output file current step number and the number of apices in the model. At the end of the simulation the `End` statement is executed, which closes the output file.

5. Implementation considerations and strategies

The task of implementing L+C posed some design problems. This chapter discusses two problems:

a) what kind of tool should be created to compile the source code in L+C,
b) what data structure should be used internally to represent the L-system string?

5.1. Interpreter vs. translator

A strategic decision that must be made is what kind of approach should be taken when implementing the new modeling language. One possible approach is to write a parser for the new language. If an existing L-system-based modeling program is available (as was the case during my research), the task of adding the new features would require extending the existing parser. Another possible approach is to extend an existing C++ parser to accommodate L-system-specific constructs.

The central part of an L-system-based modeling program is the *generator*, which derives the L-system string based on the current string and the set of productions.

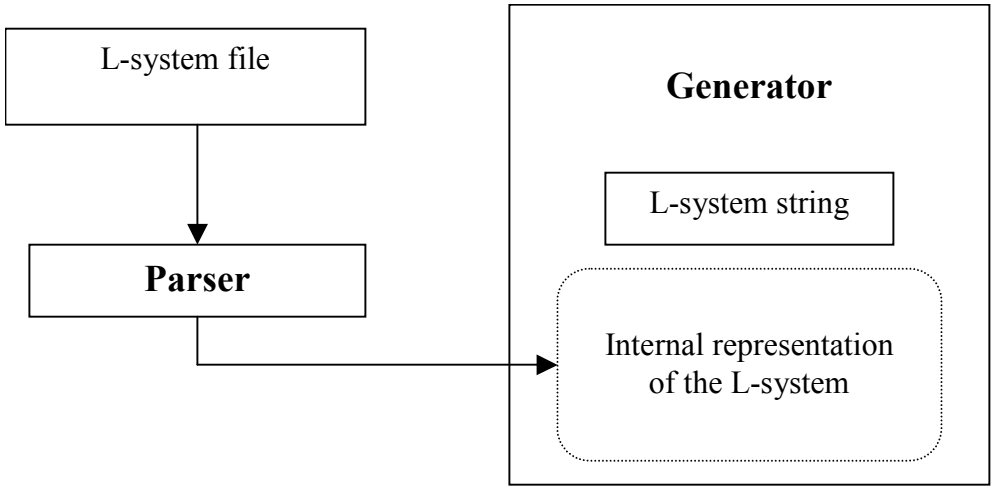


Figure 31 Parser as a module of *cpfg*

In *cpfg* modeling software, the *parser* is a component of the program. The parser takes the L-system file as its input and produces some internal representation of the L-system. In this representation data structures represent all the productions: their predecessors as well

as their code. The code (sequence of statements to be performed) is stored as a list of structures that represent individual statements; expressions are stored as arithmetic trees etc. In this approach the developer has the full control over the parser. But at the same time this approach requires the developer to write parser for elements that are not typical to L-systems but common to other programming languages: arithmetic expressions, programming statements such as assignments, loops, conditional statements, etc. Also the execution of interpreted code is known to be usually slow compared to compiled code.

A translator incorporates the following design:

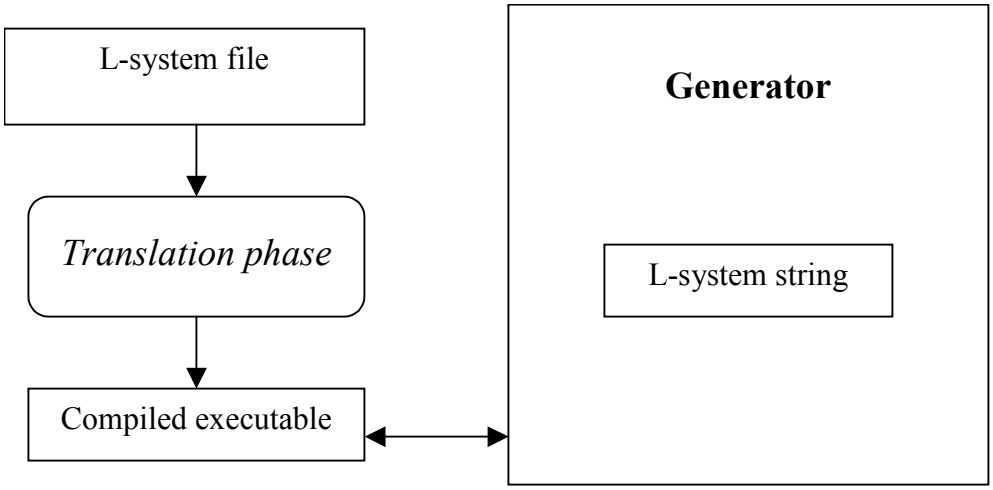


Figure 32 Schematics of the new design

For practical purposes the L-system file is not compiled into a standalone program, but into a DLL (dynamic-link library), to which a modeling program can connect at run-time. The translation phase can be designed to consist of the following steps:

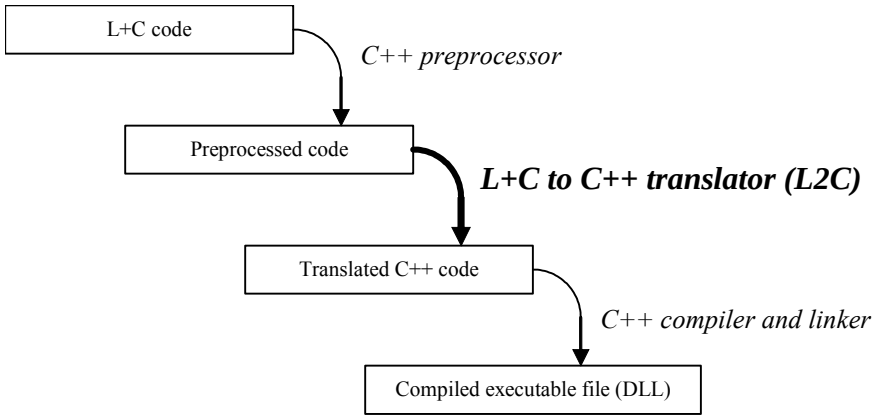


Figure 33 From L+C to compiled executable file, phases of translation

The second step – translating from the pre-processed L+C code to valid C++ code –is of main interest here, because all the other steps can be accomplished using existing tools (C++ preprocessor and compiler). The L+C to C++ translator will be also called L2C.

The main advantages of the translator approach is:

- only L+C specific elements must be identified and replaced with equivalent code in C++,
- the translator can pass verbatim all other elements of the program to the C++ compiler,
- the compiled code can be expected to execute faster than interpreted code, especially in domains in which the interpreted code is particularly slow (for example, numerical calculations).

Before the specifications of the translator can be defined fully it is necessary to discuss the problem of representing an L-system string internally.

5.2. L-system string representation

5.2.1. Traditional approach

Hanan [Han1992] proposed that the parametric L-system string be represented verbatim. In general a module with numerical parameters is stored internally in the form presented in Figure 34.

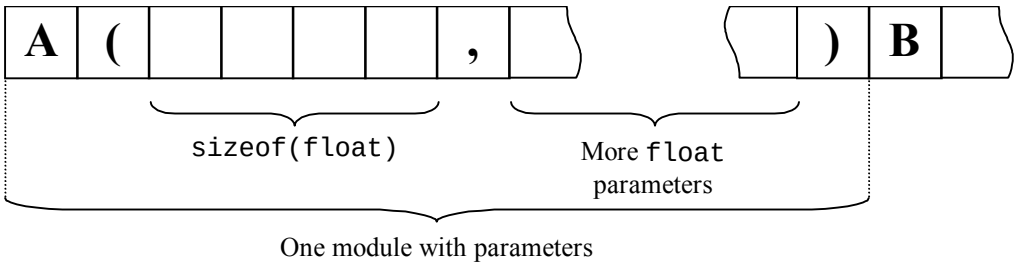


Figure 34 Traditional memory representation of L-system string

In Figure 34, every frame represents one byte. To represent a module `A` with some numerical parameters the first byte will be letter `A` (or actually the number that corresponds to the letter A in ASCII code). The next byte contains the left (opening) parenthesis. The actual parameters are stored as their binary representation in the following bytes. If the numerical parameters are of type float stored in the IEEE format they occupy four bytes (or in general `sizeof(float)` bytes). Consecutive parameters are separated with bytes containing comma '`,`'. The last parameter is followed by the right (closing) parenthesis. If a module doesn't have any parameters then its symbol is immediately followed by the next module. In addition it is assumed that the last module is followed by the null character and that the pointer to the beginning of the string is known.

To perform string derivation it is necessary to iterate forward and backward through the string – find the next and the previous module (if there is one).

The internal representation described above allows such iterations (for the actual listings see B.1). Figure 35 presents the corresponding algorithms:

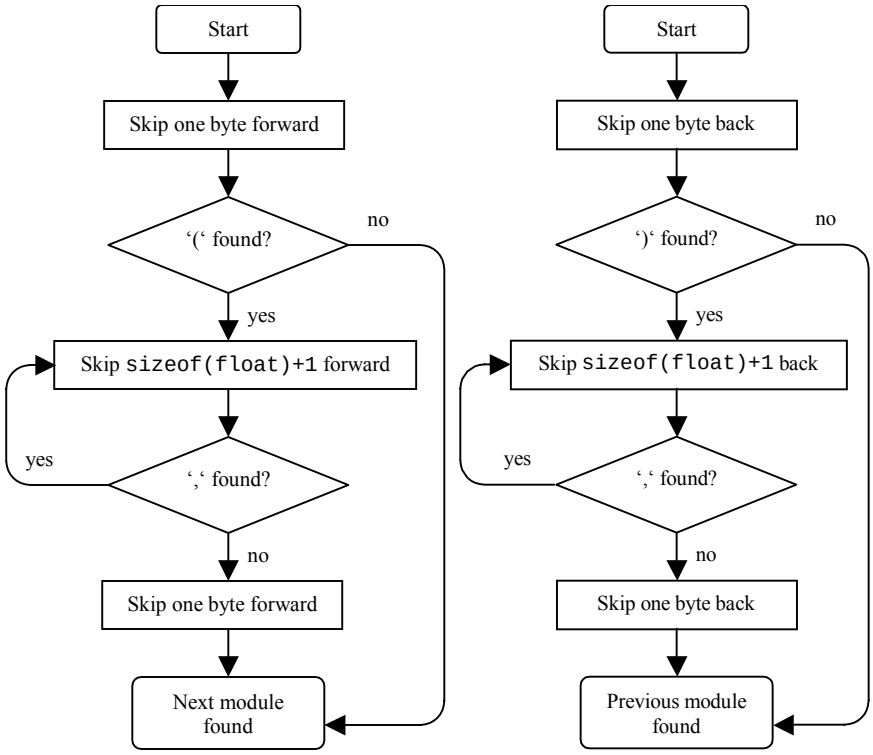


Figure 35 Algorithms to find the next and previous module for the traditional string representation

An important question is whether this representation can be extended to support modules that have parameters of user-defined types. A naïve approach could be to store user-defined type parameters in the same way as implemented previously:

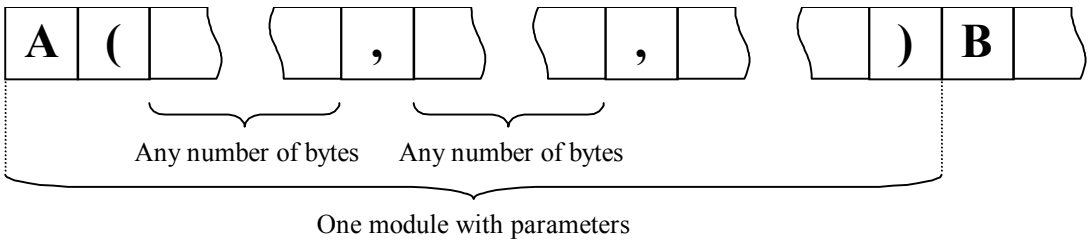


Figure 36 New L-system string memory representation, attempt one

When trying to formulate algorithms for iterating the string the following problems arise:

1. The size of parameters is not fixed. Parameters can be of arbitrary type, so they can also be of arbitrary size. Therefore it is impossible to skip a parameter forward to check whether it is followed by a comma (indicating presence of another parameter

following the comma) or the closing parenthesis. This problem might be addressed if the sizes of parameters for a given module are known.

2. When iterating backwards (searching for the beginning of the previous module) information about the sizes of parameters is not enough for two reasons. First, the size information is not available until the module has been identified. Second, a module cannot be identified until the parameters have been skipped to read its name.

5.2.2. Proposed solution

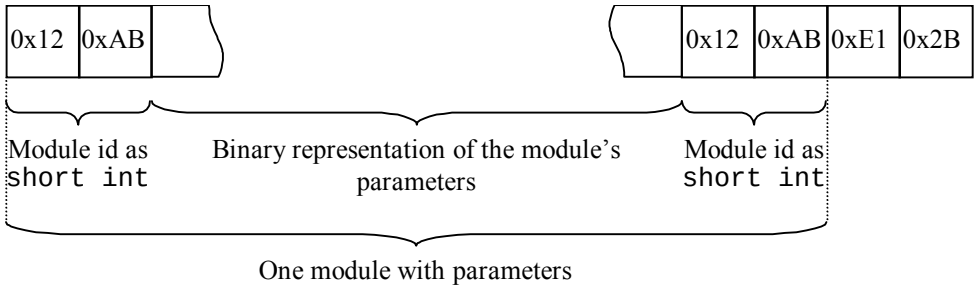


Figure 37 New L-system string memory representation

The proposed solution is presented in Figure 37. The main differences between the previous and new representation are:

1. the module's name is encoded as a fixed size identifier. Here it is assumed to be a C `short int` – an integral type stored in two bytes,
2. the identifier is present at both the beginning and end of the module,
3. parameters are not comma separated. They occupy a continuous region of memory,
4. if a module does not have any parameters it is represented as its identifier of type `short int`,
5. the format does not provide an *end-of-string* character. Information about the total length is stored separately.

With this representation the string can be iterated both forward and backward if access to the total size of parameters for every module type is provided.

Listing 46 and Listing 47 in appendix B contain the code that moves the current pointer in the L-system string to the next and previous module. The code in Listing 46 and Listing 47 refers to function `GetParametersSize`. This function is part of the *interface* that

communicates between the dynamic part of the program (L-system) and fixed generator. This interface defines how the generator manipulates the string, what code should be generated by the L+C to C++ translator and how this code should cooperate with the generator. The interface is presented in the next chapter.

6. The L+C to C++ translator

When designing the translator it is crucial to identify its top-level requirements:

1. the translator must generate valid C++ code
2. the code generated by the translator must conform to guidelines that allow for the generator to access the information required to perform production matching as well as other information specific to L-systems (derivation length, axiom, etc.)

The first requirement is obvious and requires no further explanation. The second requirement demands that some sort of *interface* be defined to link the translated (and compiled) L+C program with the generator.

The L+C translator divides its input (code in L+C) into three categories (see Figure 38):

1. Code that needs to be translated,
2. Code that requires additional code to be generated (bridge code),
3. C++ code that is not modified and passed verbatim to the C++ compiler.

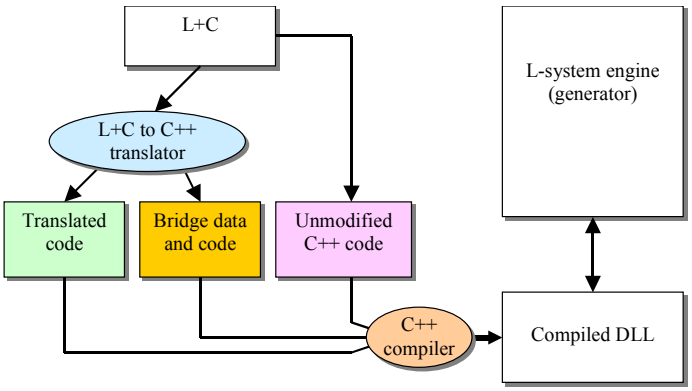


Figure 38 Relation between the components: code in L+C, L-system generator and compiled DLL.

The elements of an L+C program that are specific to L-systems and need to be translated are:

1. Global L-system parameters (derivation length, maximum decomposition depth, maximum interpretation depth),
2. Axiom,
3. Control statements,
4. Definitions of modules,

5. Productions,
6. Consider/ignore statements.

Figure 39 presents a sample code in L+C. Translation units belonging to different categories are highlighted. The following sections discuss in more detail how different statements are translated into C++ code.

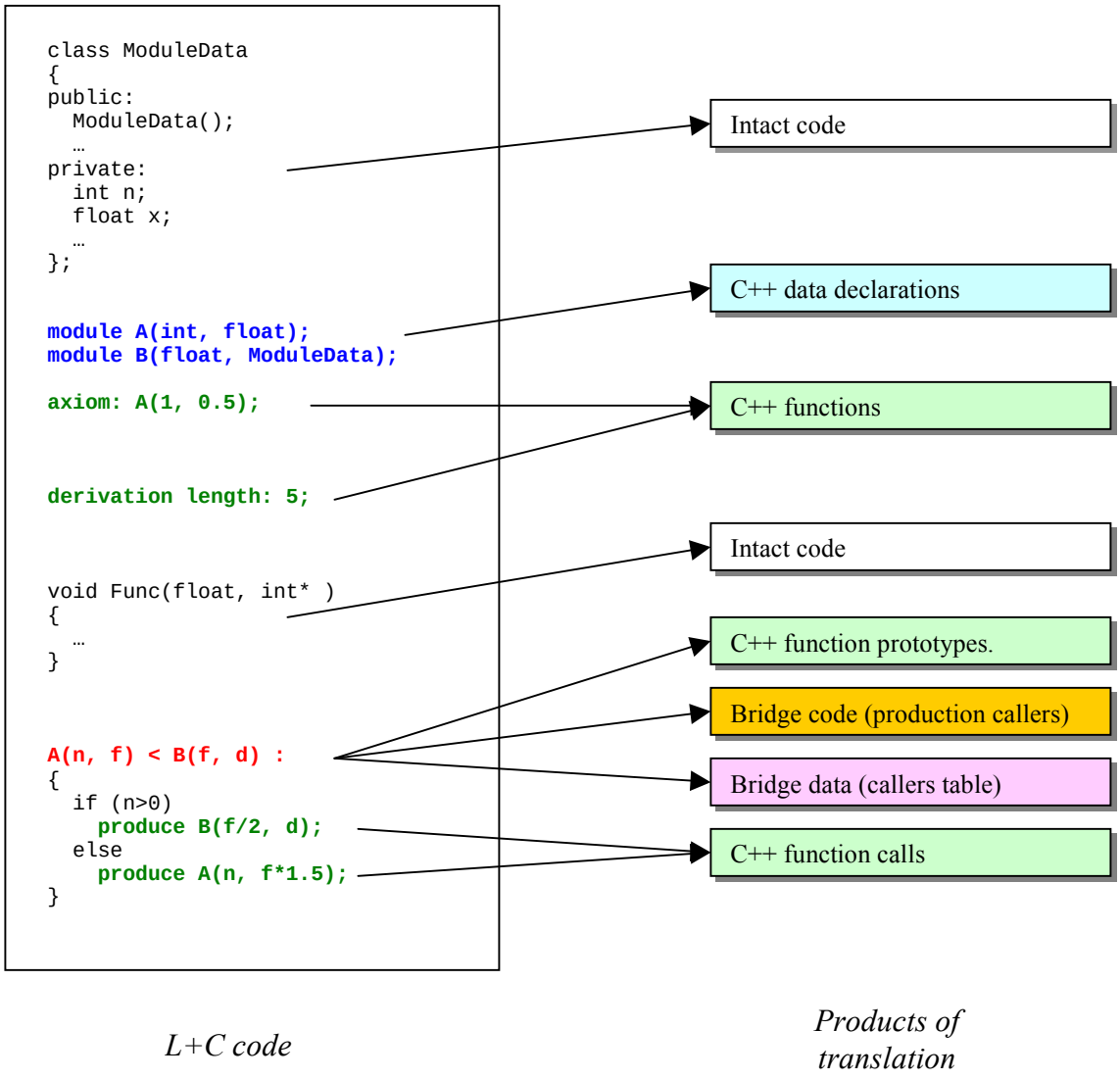


Figure 39 Sample source code in L+C, L+C to C++ translation units

The following sections identify different elements of programs written in L+C and explain how they are translated by the L2C translator and then used by *lpfg*.

6.1. Top level parameters and statements

This `Execute` function is the core of the *generator* component of *lpfg*.

Listing 31 Function executing L+C program

```
void Execute()
{
  Start();
  Axiom();
  DecomposeString(MaximumDecompositionDepth());
  for (int i=0; i<DerivationLength(); ++i)
  {
    StartEach();
    Derive();
    DecomposeString(MaximumDecompositionDepth());
    EndEach();
  }
  End();
}
```

This simplified code refers to functions, highlighted in bold, that are not part of the generator. These functions are created by the L2C translator and provide information from the L-system or perform actions specified in the L-system.

- `Start()` executes the `Start` control statement,
- `Axiom()` initializes the L-system string ,
- `MaximumDecompositionDepth()` returns the value of an expression that specifies maximum decomposition depth in the L-system,
- `DerivationLength()` returns the value of an expression that specifies the number of derivation steps to be performed as specified in the L-system,
- `StartEach()` executes the `StartEach` control statement,
- `EndEach()` executes the `EndEach` control statement,
- `End()` executes the `End` control statement.

The following sections discuss how these functions are created during translation from L+C to C++.

6.2. L-system global parameters

Every program in L+C defines the following numeric parameters:

- Derivation length
- Maximum decomposition depth
- Maximum interpretation depth

These parameters are always defined. If they are not specified explicitly (maximum decomposition and interpretation depth are optional), then the translator provides default values. These global parameters are integers. They are accessible to the generator through functions. The prototypes of the functions are:

```
int DerivationLength();
int MaximumDecompositionDepth();
int MaximumInterpretationDepth();
```

Each of the three global parameters (derivation length, maximum decomposition and interpretation depth) is specified in the L+C file by a keyword (`derivation length`, `maximum depth`) followed by an arithmetic expression and terminated with a semicolon.

The translator replaces the keyword with a C++ function prototype. The expression is copied verbatim and the closing (right) curly brace is appended. Here the replaced elements are printed in boldface while the elements copied verbatim are in italic.

Original code:	Translated code:
`derivation length:` *`i+3;`*	**`int DerivationLength() { return`** *`i+3;`* `}`

Analogous substitutions are done in case of maximum decomposition and interpretation depth.

6.3. L-system control statements

L-system control statements are statements that are executed at specific points of the simulation, and they are: `Start`, `End`, `StartEach`, `EndEach`. These statements are actually procedures without parameters. In C++ such procedures are functions that take no

parameters and return void. Because the body of the control statements can contain any valid C++ code, the L2C translator must only replace control statement keywords with C++ function prototypes to create a valid C++ function definition:

Original code:	Translated code:
Start: *{* *…* *}*	**void Start()** *{* *…* *}*

Analogous substitutions are made for the other control statements.

6.4. Module declaration

A module declaration contains two pieces of information: the name of the module and the parameters' types. The code in Listing 31 does not use any of this information explicitly. From the discussion on the internal representation of the L-system string (see section 5.2) it is known that information about the total size of the parameters is required to iterate over the string and carry out the derivation. From the same discussion it is also known that the modules in the string are not identified by their names, but by numerical identifiers. The L2C translator replaces module declarations with the declarations of module identifiers:

Original code:	Translated code:
`module A(int, float);`	`short int A_id = 101;`

The identifier *name* is created by appending _id to the module's name. The identifiers' values are consecutive integers.

This substitution does not contain information about the size of parameters of the module. Instead, size information is stored in an array, which is generated after the L+C source file has been parsed:

Listing 32 Array moduleData is generated based on the module declarations

```
struct ModuleData
{
  char* name;
  int size;
};

ModuleData moduleData[] =
{
…
  { “A”, sizeof(int)+sizeof(float) },
…
};
```

The index of every element in the `moduleData` array is equal to the module's identifier value. In the above example the entry for module `A` has index 101, which is the value of `A_id`. The module's name included in the array `moduleData` is not used during the execution of the L-system program. It is included for debugging purposes only.

6.5. Productions

Analysis of the process of string rewriting best illustrates how productions are translated, what kind of information is generated by the translator and how this information is used by the generator.

The `Execute` function in Listing 31 calls the `Derive` function. `Derive` performs the actual derivation based on the set of productions specified in the L-system. In the L+C programming language, each step of string rewriting can be performed: forward (from left to right) or backward (from right to left), which requires two versions of the function `Derive`. But these functions use the same interface to communicate with the code generated from the L+C source, so it is enough to analyze only one case. Function `Derive` presented in Listing 33 performs the derivation forward.

Listing 33 Function Derive

```
void Derive()
{
  targetstring.Clear();
  for (LstringIterator iterator(lsystemstring);
       !iterator.AtEnd(); ++iterator)
  {
```

```
    bool applied = false;
    for (int i=0; i<NumOfProductions(); ++i)
    {
      const ProductionPredecessor& predecessor =
        GetPredecessor(i);
      CallerData cd;
      if (TestMatch(iterator, predecessor, cd))
      {
        Production p = GetProduction(i);
        if (p(cd))
        {
          applied = true;
          break;
        }
      }
    }
    if (applied)
      targetstring.Append(successorStorage);
    else
      targetstring.Append(iterator.CurrentModule());
  }
}
```

The elements of the L-system – generator interface present in function `Derive` are:

- `NumOfProductions()` returns the number of productions specified in the L-system.
- `GetPredecessor(int)` returns data structure of type `ProductionPredecessor` that represents the predecessor of a production.
- `CallerData` is a data structure. It provides productions with the values of their formal parameters. It is initialized by function `TestMatch`, which determines whether the production described by `predecessor` matches the current position in the string pointed to by the `iterator`.
- `Production` is a pointer-to-function type. Functions pointed to by variables of type `Production` represent actual productions.
- `GetProduction(int)` returns a pointer to a function of type `Production` that is called to execute a production.
- `successorStorage` is a data structure that stores the modules produced by productions. If a production is applied the contents of `successorStorage` are added at the end or at the beginning of the new string, depending on the direction of derivation (see Figure 42).

Below is a detailed description of the interface elements present in Listing 33.

NumOfProductions

Function `int NumOfProductions()` is generated by L2C after the source file has been parsed. As the translator parses the L+C program it counts the number of productions and can generate the function:

```
int NumOfProductions()
{ return 7; }
```

ProductionPredecessor, GetPredecessor

Before the structure `ProductionPredecessor` can be defined it is necessary to identify what information is necessary to determine if a given production matches the current module. Matching a production requires a comparison of modules in the string and modules in the production's predecessor. The modules in the string are identified by numbers (see section 5.2). This means that a production's predecessor is fully defined by three sets of module identifiers: one set each for left context, the strict predecessor and the right context.

```
struct ProductionModules
{
  short int module_ids[maxModules];
  int count;
};

struct ProductionPredecessor
{
  ProductionModules lcntxt;
  ProductionModules strict;
  ProductionModules rcntxt;
};
```

L2C translator generates an array of structures of type `ProductionPredecessor`. Elements of this array are returned by the function `GetPredecessor(int)`.

For example, if a program contains a production with the following predecessor:

```
A() B() < B() > D() : …
```

the translator will generate the following entry in the production predecessors array:

```
ProductionPredecessor predecessors[] =
{
…
  {
    { { A_id, B_id }, 2 },
    { { B_id }, 1 },
    { { D_id }, 1 }
  },
…
};
```

Productions as procedures, CallerData, production callers

Two problems must be addressed to execute a production:

- there must be a function that represents the production, and
- there must be a way of passing actual parameters to this function.

Prusinkiewicz and Hanan [Pru1992] noted that productions are somewhat similar to functions (or actually procedures), as they are known from imperative programming languages. The similarities are:

- A production is a piece of code,
- It takes an input: its predecessor and (optionally) parameters of the predecessor's modules,
- It has output: the successor.

The main differences are:

- Productions are not called from anywhere explicitly. The general mechanism of matching productions determines which production should be applied and when. This is a general feature of declarative programming-languages.
- Productions don't return the value in the traditional sense. Instead their output modifies the contents of the L-system string.

Consider the following example (elements specific to L+C have been highlighted with boldface):

Listing 34 Sample production with multiple successors

```
struct data
{
  float l;
  int n;
};

module A(data, float);
module B(int, float);

A(dl, xl) < B(n, a) :
{
  float x = f(n, dl.l);
  if (a>xl)
    produce B(n+1, x);
  else
    produce B(n-1, xl);
}
```

The production in Listing 34 has two modules in its predecessor. The first line of this production will be regarded as corresponding to the C++ *function prototype*. A function prototype contains the following elements:

- name,
- parameters,
- return type.

In L-systems there is no *return type* of productions, because they don't return values. In C++ the function name and parameters identify the function. In L-systems, on the other hand, the names of the modules in the predecessor (and the order of their appearance in all the three parts of the predecessor: left context, strict predecessor and right context) identify a production. The parameters are implicit, the modules have to be declared beforehand.

A production predecessor is a distinctive element of L+C. On one hand, it identifies the part of the string that is to be replaced (the strict predecessor) and optionally the context – the neighbour modules of the strict predecessor. On the other hand, the production predecessor corresponds to a procedure (or function) prototype: it specifies the input

parameters of the production. The translator's task is to replace the production predecessor with a syntactically valid prototype of a function:

Listing 35 translation of a production predecessor into a function prototype

Original code:

```
A(dl, xl) < B(n, a)
```

Translated code:

```
void P1(bool& res, data dl, float xl,
int n, float a)
```

This substitution creates a valid C++ function prototype. The function name is of the form `Pnn`, where `nn` is the ordinal number of the production being translated. All the parameters have the same names as in the production predecessor (the meaning of the first parameter `res` will be explained soon). It is important to notice, that information concerning which parameters belong to which modules is lost. Are `dl`, `xl`, `n` and `a` all parameters of a single module in the strict predecessor? Maybe `dl` is a parameter of a module in left context, `xl` and `n` are parameters of a module in the strict predecessor and `a` is a parameter of a module in the right context? This information cannot be deduced from the function prototype. But the modules in every component of the predecessor are known because they are present in the `predecessors` array. Therefore it is possible to reconstruct which parameters of function `P1` correspond to which module.

The remaining problem that needs to be solved is a way of passing actual parameters from the generator to the production. Obviously the generator cannot call a production directly because it doesn't know its prototype. So there must be a unified way of passing parameters to productions regardless of their prototypes.

To address this problem *production callers* are introduced. Production callers are a bridge between the generator and productions. Production callers have fixed prototype:

```
bool (*Production)(CallerData*);
```

The production callers that are actually returned by `GetProduction(int)`. The return value of a production caller indicates whether the production was applied (if the `produce` statement was executed). `CallerData` is a structure, which contains pointers to the modules

in the string being rewritten (see Figure 40). `TestMatch` initializes `CallerData` while determining whether a given production matches the current position in the string.

Production callers are functions generated by the translator and their responsibility is to extract the parameters from the string and pass them to the actual production. Production callers also inform the generator whether the production was applied. A sample production caller corresponding to the production from Listing 35 looks as follows:

Listing 36 Sample production caller

```
bool PC1(CallerData* pCD)
{
  // extract parameters for the left context
  data dl;
  const char* pX = pCD->lcntxt.Addr[0];
  // first data
  memcpy(&dl, pX, sizeof(data));
  // skip sizeof(data) bytes
  pX += sizeof(data);
  float xl;
  // and extract a float
  memcpy(&xl, pX, sizeof(float);
  // now extract parameters for the strict predecessor
  int n;
  pX = pCD->strict.Addr[0];
  memcpy(&n, pX, sizeof(int));
  pX += sizeof(int);
  float a;
  memcpy(&a, pX, sizeof(float));
  // if there were any modules in the right predecessor
  // the parameters would be extracted here
  bool res = false; // assume that the production did not apply
  // call the production-function
  P1(res, dl, xl, n, a);
  return res;
}
```

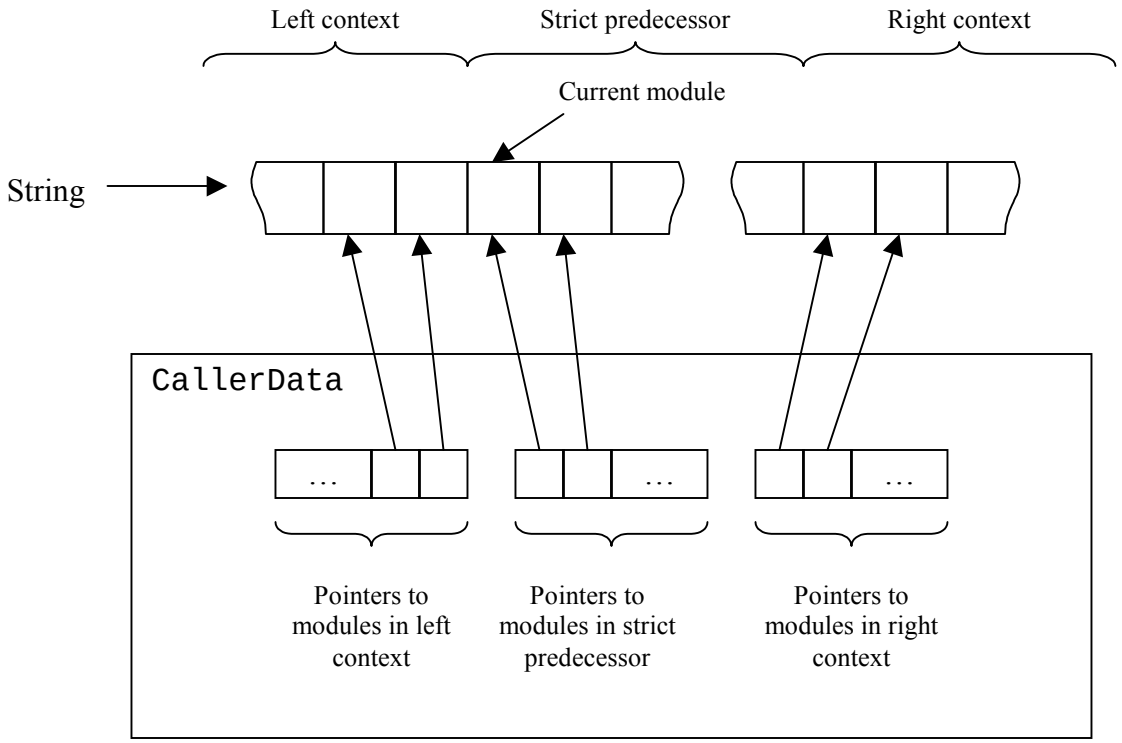


Figure 40 `CallerData` makes it possible to access a production's actual parameters

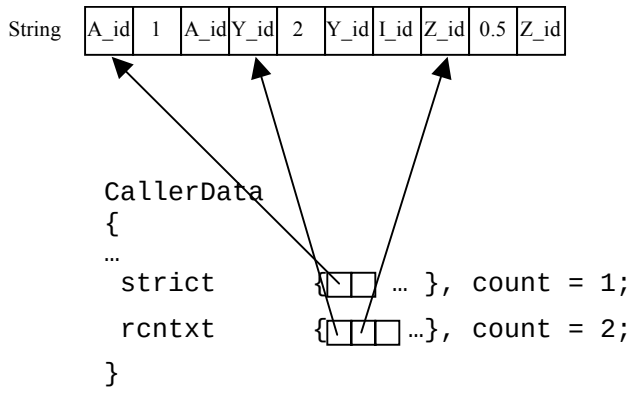


Figure 41 Mapping parameters locations into a `CallerData` structure

The actual definition of `CallerData` is as follows:

```
struct ActualParameters
{
  char* Addr[MaxParameters];
```

```
  int count;
};

struct CallerData
{
  ActualParameters lcntxt;
  ActualParameters strict;
  ActualParameters rcntxt;
};
```

Successor storage

Modules generated by the produce statement are not added to the string immediately but are first stored in a data structure called *successor storage*. The reason for this is that the modules in the successor are always generated in order from left to right. When the string is being derived from right to left, the modules should be added in the reverse order to that in which they were created. There are two possible approaches: one is to change the order in which the elements of the successor are generated, depending on the direction of derivation. The other is to use an intermediate buffer – once the successor is created it is added to the string (in front or at the end depending on the derivation direction).

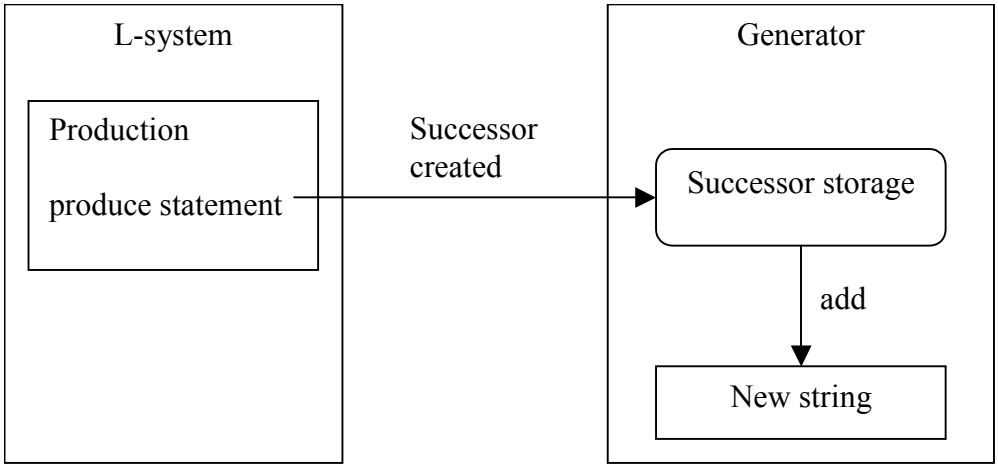


Figure 42 Modules generated by productions are first stored in the *Successor storage*, then transferred to the new string.

The first solution complicates the code generating the successor. The second involves some time overhead – an additional copy operation. In the current implementation the second solution was chosen. Profiling indicates that the use of the successor storage accounts for about 0.5% to 2% of the program's run-time, which judged to be acceptable.

6.6. *The* `produce` *statement*

The `produce` statement plays two roles, specifying the successor of the production, and implying that the execution of the production should terminate (`return` statement)[10]. While building a successor, the DLL generated from the L+C source file needs to modify data that belongs to the main program.

`SuccessorStorage` is the data structure where the successor is created by a production. After the production is applied, the complete successor is added to the string. The generator has to provide a function that will let the DLL (or the productions to be exact) add data to the `SuccessorStorage`. This function is called `Add`:

```
void Add(const void*, int);
```

The first parameter is a pointer to the data to be added and the second parameter is the number of bytes to be added. The code calling `Add` takes full responsibility that the data stored in the `SuccessorStorage` are correct, e.g. it follows the specifications of the L-system string. If the data to be added are contained in variables, the `Add` function could be used directly:

```
Add(&B_id, sizeof(short int));
Add(&n, sizeof(int));
Add(&x, sizeof(float));
Add(&B_id, sizeof(short int));
```

But in general this is not the case. Both `produce` statements in Listing 34 generate modules with their parameters specified using expressions. C++ does not allow one to obtain the address of an expression. Consequently the following construct is not valid in C++:

---

[10] The two functions could be split, so that the keyword `produce` just *produced* the successor, but did not terminate the production. Alternatively another keyword (for example `insert`) could be introduced to generate the successor, but not terminating the production, making it possible to build the successor in a series of `insert` statements. This idea seems worth considering in the future research.

```
Add(&(n+1), sizeof(int));
```

One possible solution is to declare local variables that store the values of parameters and use them in the call to `Add`:

```
Add(&B_id, sizeof(short int));
int n1 = n+1;
Add(&n1, sizeof(int));
Add(&x, sizeof(float));
Add(&B_id, sizeof(short int));
```

This approach is acceptable but would complicate the implementation of the L+C parser.

A more effective and general solution has been chosen. A template function `Produce` is defined as follows:

```
template<class T>
void Produce(T t)
{ Add(&t, sizeof(T)); }
```

Now the translation of the `produce` statement is straightforward:

```
Produce<short int>(B_id);
Produce<int>(n+1);
Produce<float>(x);
Produce<short int>(B_id);
```

This solution takes advantage of the C++ compiler's optimizing capabilities and greatly simplifies both the L+C parser and the generated code. Note the explicit types in the instantiation of `Produce`, so that the type used is the same as the type of the parameter being produced. Otherwise the C++ compiler could perform an implicit type conversion and generate `Add` for a wrong type. For example, it is common to use a literal like `1` where a `float` is expected instead of `1.0f`. But in this case the compiler would assume that `<int>` instance is required. This kind of mistake could result in a corrupted string.

Finally, the production caller must be informed if the production was actually applied. This is done by assigning `true` to the parameter `res` (see Listing 35 and Listing 36). Finally all the code generated in place of the `produce` statement is enclosed in curly braces, so that it actually forms a single (compound) statement:

```
{
  Produce<short int>(B_id);
```

```
    Produce<int>(n+1);
    Produce<float>( x);
    Produce<short int>(B_id);
    res = true;
    return;
  }
```

6.7. Other elements

6.7.1. Ignore, consider

The `ignore` and `consider` statements are mutually exclusive. Each of these statements is replaced by a declaration of an array of module identifiers and a function that returns the number of elements in the array. For example:

Original code:

```
ignore: A B Left Right;
```

Translated code:

```
short int ignored[] =
{ A_id, B_id, Left_id, Right_id };
int NumOfIgnored() { return 4; }
```

6.7.2. Axiom

Axiom is a special type of production that does not have a predecessor. It is replaced by a function named `Axiom`. Contents of the axiom are expanded as if it were a `produce` statement. This function can be called directly by the generator.

Original code:

```
Axiom: F(1) Left(90) F(1);
```

Translated code:

```
void Axiom()
{
  Produce<short int>(F_id);
  Produce<float>(1);
  Produce<short int>(F_id);
  …
}
```

6.7.3. Production, decomposition, interpretation

These keywords are not actually translated. Instead, they change the internal state of the translator that determines the allowed types of the productions (if context-sensitive productions are allowed). Also, in the case of decomposition and interpretation rules there is no array of `ProductionPredecessor`. Instead there are two arrays of `ProdCallers`: one for interpretation and one for decomposition rules.

The specifications of the translator presented in this chapter can be summarized in a form of a diagram. Figure 43 shows how L+C input is divided into standard C++ code and L+C specific code. The L+C constructs are translated into C++ declarations and definitions depending on their type.

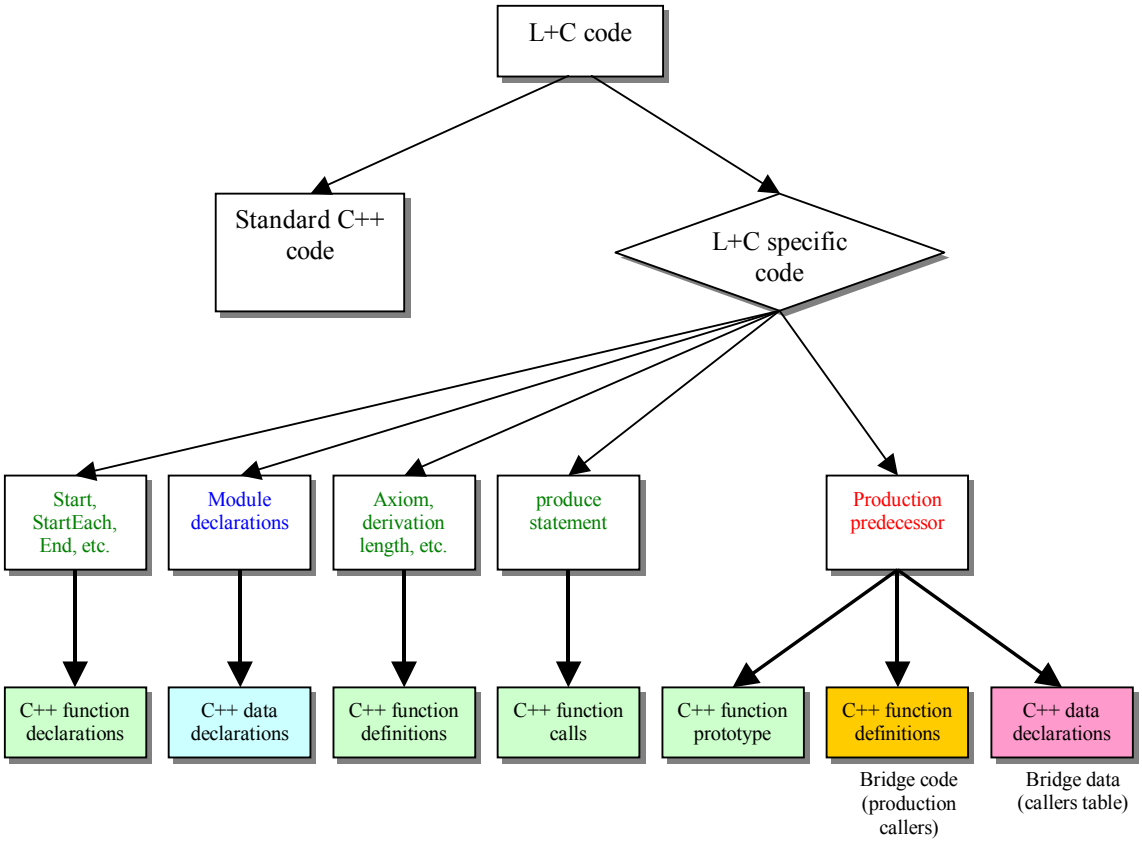


Figure 43 L+C to C++ translator, translation units

7. Application examples

This section presents examples that demonstrate the use of the L+C modeling language. They have been selected to demonstrate the use of concepts and features introduced in L+C.

7.1. Model of Anabaena

This example implements a developmental model of a filamentous cyanobacterium *Anabaena catenula*. The distribution of heterocysts (term described below) in the filament tends to form a pattern. The model captures this tendency by operating on genes' expression: production of two proteins. This model extends the model presented in Listing 1 in section 2.1 and was written by P. Prusinkiewicz (unpublished). It employs user-defined structures to store the parameters that describe cells and user-defined functions to perform calculations.

In the model presented in Listing 37, the cells are characterized by the following parameters:

- Concentration of protein *hetR*,
- Concentration of protein *patS*,
- Length,
- Polarity,
- Differentiation degree.

Initially the model consists of two cells. During the simulation the concentration of the activator *hetR* and inhibitor *patS* controls cell development. The concentration changes as the result of the following processes:

1. diffusion of patS (transfer of the substance from a cell of high concentration to cells of lower concentration),
2. production,
3. decay.

Diffusion

The change of concentration caused by diffusion of *patS* from a cell of size *x* over the time increment Δt is expressed by formula:

$$\Delta patS_{diff} = D\left(\frac{patS_2 - patS}{xw}\right)\Delta t\ ,$$

where *D* is diffusion constant, *w* is the width of the cell, and $patS_2$ is the concentration in the neighbouring cell. If the concentration in the current cell is greater than that in the other cell, then the substance will diffuse to the other cell and the concentration will decrease (the numerator $patS_2$-*patS* is negative). If the concentration in the current cell is lower it will increase.

If the current cell has two adjacent neighbours (labelled *l* and *r*), diffusion equals the sum of the diffusions between the cell and both of its neighbours. Diffusion is then expressed by formula:

$$\Delta patS_{diff} = D\left(\frac{patS_l + patS_r - 2\,patS}{xw}\right)\Delta t$$

Production

The amount of *patS* produced in the time interval Δt depends on the concentration of *hetR* in the cell. It is expressed by the formula

$$\Delta patS_{prod} = \rho\left(\frac{hetR^2}{1+\kappa\cdot hetR^2} + h_0\right)\frac{\Delta t}{x},$$

where ρ, κ and h_0 are parameters, x is the size of the cell. *HetR* acts as the activator.

The amount of *hetR* during the time interval Δt depends on the concentration of both *hetR* and *patS*. *HetR* is an activator and *patS* is an inhibitor as expressed by the formula below:

$$\Delta hetR = \frac{\rho}{patS}\left(\frac{hetR^2}{1+\kappa \cdot hetR^2} + a_0\right)\frac{\Delta t}{x}$$

If the concentration of hetR exceeds a threshold, the cell becomes a heterocyst. Heterocysts do not divide.

Decay

The decay of proteins during Δt is expressed by the following formulas:

$$\Delta hetR = -\mu\, hetR\, \Delta t \qquad \Delta patS = -\nu\, patS\, \Delta t\,,$$

where μ and ν are decay parameters. Decay is implemented in the decomposition rule d_1.

Growth and division

During every time step cells grow in length (decomposition rule d_1), resulting in decreased concentration of *hetR* and *patS*. Cells grow during the simulation and divide (except for heterocysts) if their size exceeds a threshold. Polarity of the cells determines which of the daughter cells is longer.

Implementation of the model

Modules of type `Cell` represent cells. The information associated with every cell is stored in the `Cell` parameter of type `CellData`.

Diffusion is controlled by productions p_1, p_2 and p_3. p_2 and p_3 apply when a cell has only one neighbour. During the derivation, diffusion of *patS* is calculated. The successors of the productions are modules of type `TempCell`, to distinguish between the cells before and after the growth phase handled by decomposition. This distinction avoids infinite recursion in the decomposition rules.

The growth phase of the simulation (decomposition d_1) involves three stages. First, the new concentrations of both *patS* and *hetR* are determined as a result of their production by the cell using the `HetR` and `PatS` functions. Then the decay of the substances is calculated.

Finally, the cells grow and those that reach the maximum size and are not heterocysts divide.

Listing 37 Model of Anabaena in L+C

```
enum Polarity
{ plRight, plLeft };

// daughter cell length coefficients
const float longer = 0.55f;
const float shorter = 1.0f - longer;

const float rho = 3;         // protein production parameter

const float a0 = 0.01;       // base activator production
const float h0 = 1;          // base inhibitor production

const float mu = 0.1;        // hetR decay rate
const float nu = 0.45;       // patS decay rate
const float D_patS = 0.0045; // diffusion coefficient
const float kappa = 0.001;   // protein production coefficient
const float dt = 0.5;        // time step
const float w = 0.01;        // diffusion w parameter
const float lm = 1.0;        // cell maximum size
const float gr = 1.002;      // growth rate of cells
const float thr = 0.5;       // treshold hetR value for heterocysts

struct CellData
{
  float hetR, patS; // protein concentrations
  float x;          // cell size (length)
  Polarity  p;      // plLeft or plRight
  float vph;        // differentiation degree
                    // vegetative-pro-hetero
};

// Definition of modules used in the model
module Cell(CellData);
module TempCell(CellData);

// Production of proteins
float HetR(float hetR, float patS)
{
  return rho/patS*(hetR*hetR/(1+kappa*hetR*hetR) + a0);
}

float PatS(float hetR)
{
  return rho*(hetR*hetR/(1+kappa*hetR*hetR) + h0);
}

// Parameters of the initial two cells
CellData icd1, icd2;
```

```
Start:
{
  icd1.hetR = 0.1;
  icd1.patS = 100.0;
  icd1.x = longer;
  icd1.p = plRight;
  icd1.vph = 0;

  icd2 = icd1;
  icd2.x = shorter;

}

Axiom: Right(90) Cell(icd1)Cell(icd2);

// Interaction between cells (diffusion of patS)
p1: Cell(cdl) < Cell(cd) > Cell(cdr) :
{
  cd.patS += (D_patS*(cdl.patS+cdr.patS-2*cd.patS)/(cd.x*w))*dt;
  produce TempCell(cd);
}

p2: Cell(cd) > Cell(cdr) :
{
  cd.patS += (D_patS*(cdr.patS-cd.patS)/(cd.x*w))*dt;
  produce TempCell(cd);
}

p3: Cell(cdl) < Cell(cd) :
{
  cd.patS += (D_patS*(cdl.patS-cd.patS)/(cd.x*w))*dt;
  produce TempCell(cd);
}

decomposition:

d1: TempCell(tcd) :
{
  CellData cd = tcd;

  // Proteins production
  cd.hetR += HetR(tcd.hetR,tcd.patS)*dt/cd.x;
  cd.patS += PatS(tcd.hetR)*dt/cd.x;

  // Decay of proteins
  cd.hetR -= mu*tcd.hetR*dt;
  cd.patS -= nu*tcd.patS*dt;

  // If did not reach max size, grow
  if(cd.x < lm)
  {
    cd.hetR /= gr;
    cd.patS /= gr;
    cd.x *= gr;
    // differentiation degree of heterocyst
```

```
    // is identified with its size
    if(cd.hetR>thr)
      cd.vph=cd.x;
  }


  // If maximum size not reached, or heterocyst, that's it
  if (cd.x<lm || cd.hetR>thr)
    produce Cell(cd);

  // otherwise, divide
  else
  {
    CellData cd1 = cd;
    CellData cd2 = cd;
    cd1.p = plLeft;
    cd2.p = plRight;
    // take polarity into account
    if (cd.p==plRight)
    {
      cd1.x *= longer;
      cd2.x *= shorter;
    }
    else
    {
      cd1.x *= shorter;
      cd2.x *= longer;
    }
    produce Cell(cd1) Cell(cd2);
  }
}

interpretation:

// Display the cell
i1: Cell(cd) :
{
  float width;
  width = max(shorter, cd.vph);
  produce SetWidth(width) f(width/2)
      Circle(width/2)F((cd.x-width)/2)
      SB()
        Right(90) SetColor(2)
        SetWidth(shorter/4)F(log(cd.patS))
      EB()
      SB()
        Left(90) SetColor(3)
        SetWidth(shorter/4)F(log(1000*cd.hetR))
      EB()
      F((cd.x-width)/2)Circle(width/2)f(width/2);
}
```

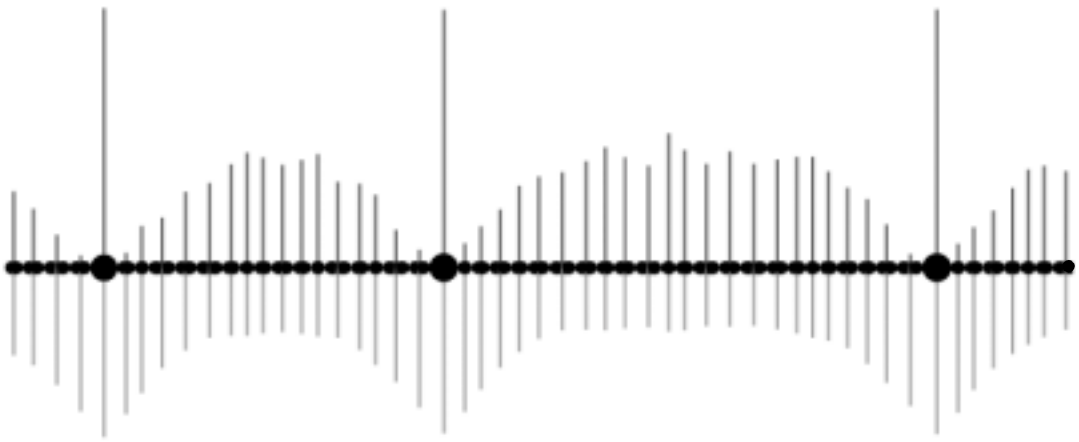


Figure 44 Image generated by the model in Listing 37

Figure 44 presents the visualization of the model. Rounded lines, with the length proportional to the size of the cell, represent the cells (interpretation i_1). Large, round cells are heterocysts. In addition the vertical lines represent the concentrations of both *patS* and *hetR* in the cells. The length of the dark lines is proportional to the concentration of *hetR*, while the length of lighter lines is proportional to the concentration of *patS*.

The ability to store parameters in a user-defined structure allows one to easily extend the model by adding new members to the structure. In parametric L-systems it would have been necessary to rewrite all the productions that involve the module `Cell` (that represents individual cells) to include additional parameters.

7.2. Borchert-Honda model

This example implements a developmental model of a branching structure presented by Borchert and Honda in [Bor1984]. It uses a user-defined type as a module's parameter; user-defined functions and fast information transfer to propagate acropetal and basipetal signals throughout the plant structure. The L+C implementation in Listing 38 is based on an L-system implementation presented in [Pru1997a].

The objective of the original model was to propose a mechanism that controls the number of branches created by a tree and prevents the exponential grow of the number of the branches.

The program operates on three types of modules: apices `A`, internodes `I` and an auxiliary module `N`. Each internode has a parameter of type `InternodeData`. `InternodeData` is a structure type that contains the following fields: segment type (`st`), flux value (`flux`) and apex count (`count`).

In this model, plant development is controlled by the amount of substances that propagate acropetally (from the base of the structure towards the apices). The process of development is modeled in discrete time steps. The development starts with a single internode and an apex. This internode is called the base of the tree. Development of the plant depends on the flux (flow of substances) available for every apex in the plant. At the beginning of the simulation the base of the tree contains the initial concentration of growth substances.

Each time step is divided into three phases. Each phase corresponds to a derivation step. Variable `stepType` of type `PhaseType` controls the current phase. Its value is changed at the beginning of every derivation step (`StartEach` statement) and cyclically assumes values BSP (basipetal signal phase – counting apices), ASP (acropetal signal phase – distributing flux) and GP (growth phase).

Growth phase

The age of the base is incremented (production p_1). The amount of growth substances available for every apex is checked. If this amount exceeds a threshold, the apex produces two new branches: main and lateral (production p_2).

Basipetal signal phase

During this phase the derivation is performed backward (see `StartEach` statement). The number of apices supported by each internode is calculated (production p_3). This includes the apex following the internode in question or apices supported indirectly by the daughter branches of the internode. The number of apices supported by internodes is considered an acropetal signal and is transferred using fast information transfer.

Acropetal signal phase

In this phase the derivation is performed forward (see `StartEach` statement). The amount of flux available for all internodes is determined. At the base the flux is calculated (production p_4) using the formula proposed by Borchert and Honda: $v = \sigma_0 2^{(k-1)\eta^k}$ (see function `BaseFlux`). This formula simulates a sigmoidal increase of flux over time. Starting from the base internode, the flux is divided between the daughter branches (production p_5). The amount of flux available for a daughter internode depends on the number of apices supported by the branch. Main branches are preferred over lateral branches in the sense that they are assigned more flux. If two branches support the same number of apices then the main branch is assigned λ (constant `lambda`) of the flux reaching the branching point and the lateral branch obtains the remainder (1- λ) of flux. If the number of apices supported by the main and lateral branch is different, then the flux reaching the lateral branch is multiplied by the ratio c/c_s, where c is the number of apices supported by the lateral branch and c_s is the number of apices supported by the main branch. The fraction of flux available for a branch is calculated in the function `Flux`.

Listing 38 Borchert-Honda model implemented in L+C using fast information transfer

```
#include <math.h>    // required for pow
#include <lpfgall.h> // predefined modules and data structures

const float Alpha1 = 10.0f; // branching angle - main segments
const float Alpha2 = 32.0f; // branching angle - lateral segments
const float sigma0 = 17.0f; // initial flux
const float eta    = 0.89f; // input flux change parameter
const float vth    = 5.0f;  // threshold flux for branching
const float lambda = 0.7f;  // flux distribution factor

derivation length: 36;

int StepNo;                 // Step number counter

enum PhaseType
{ BSP = 0, ASP, GP };

PhaseType steptype;

enum SegType
{ stBase, stStr, stLat };

struct InternodeData
{
  SegType st;
```

```
  float flux;
  int count;
};

InternodeData iBase = { stBase, 0.0, 1 };

Start: { StepNo = 0; }

StartEach:
{
  steptype = (PhaseType) (StepNo % 3);
  switch (steptype)
  {
  case BSP :
    Backward(); // Derive backward
    break;
  case ASP :
    Forward();  // Derive forward
    break;
  }
}

EndEach:
{ StepNo++; }

module A();
module I(InternodeData);
module N(int);

ignore: Left Right RollR;

// flux at the base
float BaseFlux(int age)
{ return sigma0*pow(2.0, (age-1)*pow(eta,age)); }

// flux distribution
// count – number of apices supported by the internode
// pcount – number of apices supported by the parent internode
// internode type (lateral or main)
float Flux(int count, int pcount, SegType st)
{
  if (stLat == st)
    return (1-lambda)*(1.0*count/(pcount-count));
  else // stStr == st
    return 1-(1-lambda)*(1.0*(pcount-count)/count);
}

Axiom: SetWidth(0.1) N(1) I(iBase) A;

p1: N(k) :
{
  if (steptype == GP)
    produce N(k+1);
}
```

```
p2: I(idata) < A() :
{
  if ((steptype == GP) && (idata.flux>vth))
  {
    InternodeData iLat = { stLat, idata.flux*(1-lambda), 1 };
    InternodeData iStr = { stStr, idata.flux*lambda, 1 };
    produce
    RollR(180)
    SB Right(Alpha2) I(iLat) A EB
    Left(Alpha1) I(iStr) A;
  }
}

p3: I(idata) >> SB() I(idata1) EB() I(idata2) :
{
  idata.count = idata1.count + idata2.count;
  produce I(idata);
}

p4: N(k) << I(idata) :
{
  idata.flux = BaseFlux(k);
  produce I(idata);
}

p5: I(idataL) << I(idata) :
{
  idata.flux = idataL.flux*Flux(idata.count, idataL.count, idata.st);
  produce I(idata);
}
```

The amount of flux produced at the base of the tree depends linearly on the parameter σ_0 (`sigma0`). If this value is higher there is more flux available for the lateral branches and the tree tends to grow wider. The simulation has been performed for two values of `sigma0`. The results are presented in Figure 45.

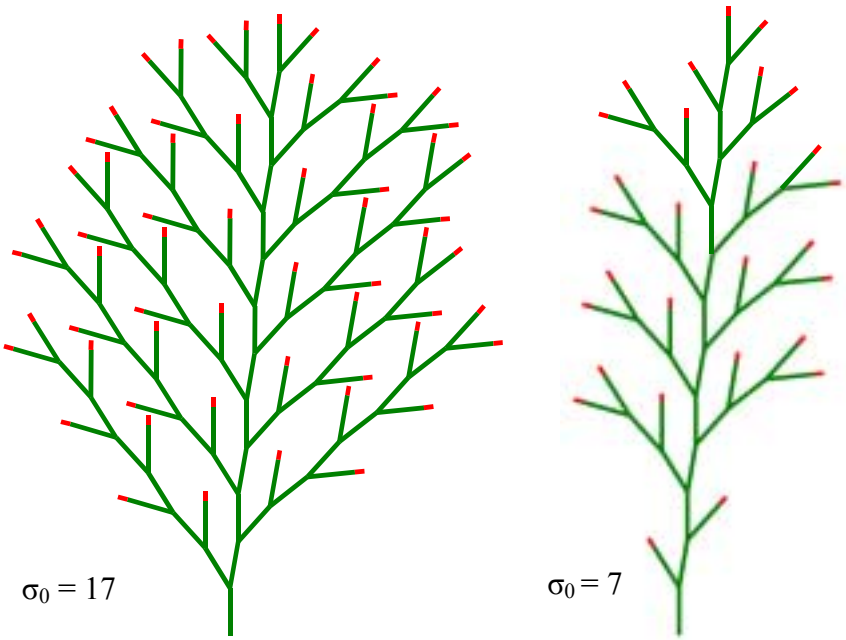


Figure 45 Two images generated by the L-system from Listing 38 for two values of σ_0

8. The L-studio modeling environment

During my research I have designed and implemented the modeling environment L-studio. Originally it was created to address the need for an L-system based plant modeling environment for MS Windows, as this operating system is more popular among biologists than Unix. Later the development of L-studio was used as an opportunity to test some new interactive and visual modeling techniques. L-studio together with a set of additional programs constitutes a plant modeling software system. The whole system consists of the following elements:

- L-system based simulation program cpfg (**p**lant and **f**ractal **g**enerator with **c**ontinuous parameters),
- L-system based simulation program lpfg (**p**lant and **f**ractal **g**enerator implementing the **L**+C language),
- L-studio modeling environment that provides visual tools and serves as a control component in the process of modeling and performing simulations
- A set of programs for simulating environmental processes that affect plant development
- A set of sample models

L-studio is specifically designed for the MS Windows operating system. It has a counterpart in the Unix system, Virtual Laboratory (*vlab*) [Mer1990, Mer1991, Fed1999]. All other components of the system (*cpfg*, *lpfg* and environmental programs) are designed to be platform-independent and they can be used under Windows as well as Unix operating systems. The portability has been achieved by the use of the C and C++ programming languages and the dependencies on any external libraries have been reduced to a minimum: graphic output is implemented using OpenGL library and user interface has mostly been entrusted to the command line options. At the same time the system provides a user-friendly way of invoking the simulation programs without the need of manually typing commands.

The L-studio modeling environment is *object* (or *model*) *oriented*. An object is a set of files that are used in a simulation. An object-oriented environment means that its components are designed to cooperate and help the user in the process of developing and experimenting with the object.

The description of L-studio contained in this chapter is not intended to be a user's manual. Instead it presents selected elements of the system. The selection is intended to give a general overview of L-studio and to present concepts incorporated in the system that I have introduced or extended. An overview of the L-studio/cpfg modeling system from the user's perspective can be found in [Pru1999]. A user's manual is available at http://www.cpsc.ucalgary.ca/Research/bmv/lstudio/index.html.

8.1. Object organization

Every object consists of a set of files. All files constituting an object are stored in a separate directory. The files that constitute a model are typically: L-system file, view parameters file and colours definition file etc. This design has been borrowed from an implementation of a prototype-extension paradigm [Lie1986] as it is found in *vlab*, although the functionality in this domain available in L-studio is limited compared to *vlab*.

When working with an object, L-studio offers the user specialized editors to manipulate different types of files. Different editors are accessible under different tabs:

Figure 46 L-studio project tabs

Almost all files controlled by L-studio are text files. Some of them are directly edited as text files by the user. These are: L-system, view parameters, description. Other text files are edited using specialized visual editors.

8.1.1. Animation parameters editor

The animation parameters editor is a form-based editor. All parameters controlling animation are specified using simple GUI controls: edit lines, check boxes, radio buttons, etc.

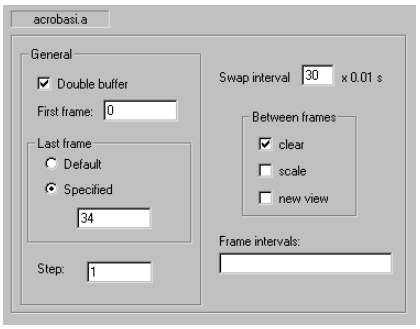


Figure 47 Animate parameters editor

Every check box and every edit line corresponds to an entry in the animate file. But the use of this form-based dialog relieves the user from the necessity of memorizing the syntax of all the options as they appear in the file.

8.1.2. Colormap editor

Images generated using *cpfg* and *lpfg* modeling programs can be rendered in one of two colour modes: colormap mode and material mode. In the L-system the current drawing colour or material is specified using an index.

In the colormap mode the drawing colour is specified as a triplet of the RGB components. All colours available for the model are stored in a palette – a set of 256 colours. These colours are manipulated using the colormap editor. To modify a colour in the palette, the user selects the colour and uses three sliders to modify the colour's components. The effect of the change is visible immediately in the palette as the selected entry is updated.

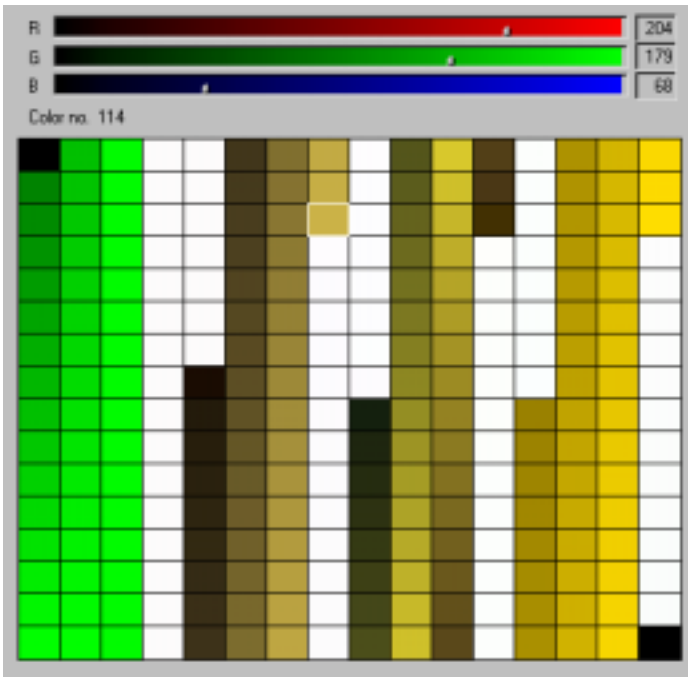


Figure 48 Screenshot of the colormap editor

8.1.3. Material editor, gallery of objects

Material rendering mode is used to create images that look more realistic than in colormap mode. In material mode, scenes are drawn using the Phong shading model as implemented by OpenGL [Woo1999]. In this model materials are specified by a set of material parameters, including four colour parameters (ambient, diffuse, specular, emission) and two numerical parameters (shininess and transparency). Each of these parameters can be modified independently.

The material editor in L-studio uses a view/edit/gallery scheme (see Figure 49). I designed this scheme to simplify editing and managing entities of the same type in a unified manner.

The gallery is an abstraction that represents a set of elements. The elements stored in a gallery are accessible through the *gallery window* where they are displayed. The way that elements are rendered in a gallery is sometimes simplified and does not contain all pertinent information (for example the gallery of panels displays only the names of the elements, see section 8.3). However elements in the gallery window must be distinguishable and recognizable. Basic operations that can be performed in a gallery are:

- Adding new elements;
- Deleting existing ones;
- Rearranging (changing the order) within the gallery;
- Copying and pasting of elements between galleries – thus moving components of L-studio objects without the necessity of manually transferring and/or editing the files where they are stored;
- Performing special operations on subsets of elements. For example, the material gallery makes it possible to interpolate colour components within a range of selected materials (see Figure 49).

Every gallery cooperates with a corresponding viewer/editor, so that the element selected in the gallery is the one being edited in the editor. When the user selects another element from the gallery, changes made to the element previously selected are first propagated to the gallery and then the newly selected element is ready for editing.

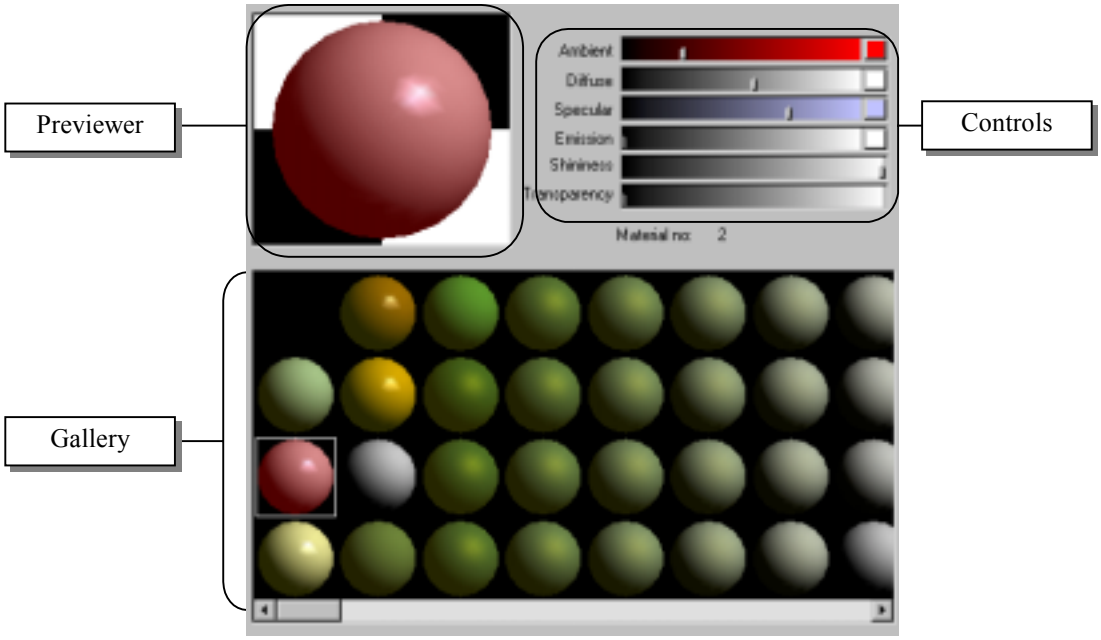


Figure 49 Screenshot of the material editor

In the material editor, the viewer/editor part consists of the material previewer window and six controls – colour sliders. All parameters defining a material can be manipulated using the colour sliders and a colour-chooser window (not shown).

8.1.4. Surface editor

Cpfg and *lpfg* make it possible to draw bicubic (Bézier) surfaces [Han1992, Fol1990]. The surface editor provides an interactive, graphical way to define and edit these surfaces.

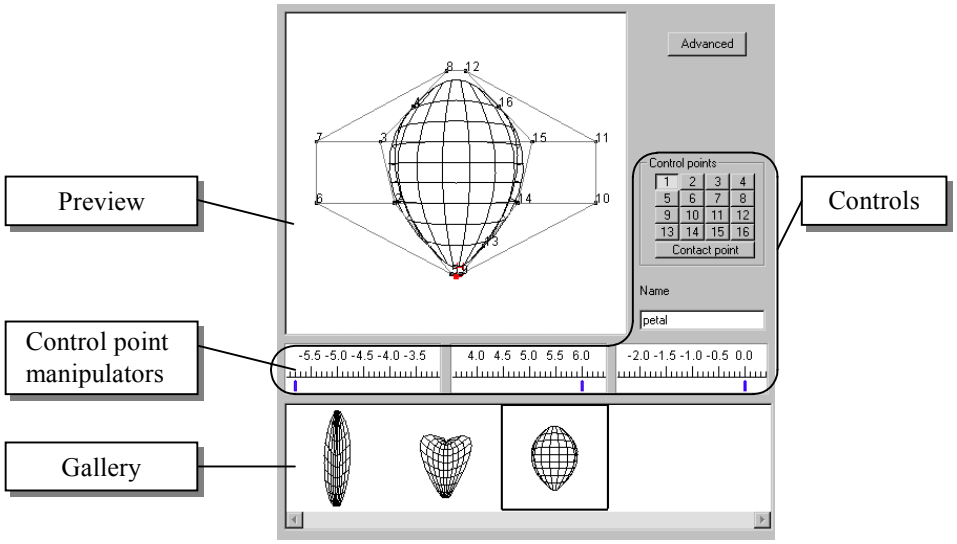


Figure 50 Screenshot of the surface editor

In the surface editor the responsibilities of the editor components (previewer and controls) are different compared to the material editor. In the material editor, the previewer is passive – it displays the current state of the element being edited. In the surface editor, elements of the gallery (surfaces), can be edited both in the preview window and using controls. To modify the surface directly in the preview window the user can click and drag control points with the mouse. To modify a surface using the controls, the user selects a control point by pressing a button (labelled 1 to 16) and changes the point's coordinates using control point manipulators located above the gallery. Control points in the preview window can be moved only in the XY plane, consequently the *z* coordinate can be modified only using the control point manipulator.

The surface gallery is an example of the gallery that displays only a subset of information related to the elements because the names of surfaces are not visible in the gallery.

8.1.5. Contour editor

The contour editor makes it possible to define two-dimensional B-spline curves [Fol1990]. The curves can be used later as cross-sections of generalized cylinders [Blo1985, Mec1997a]. The contours are modified by directly moving control points in the previewer. Controls make it possible to toggle between closed and open contours and to specify its name. The contour editor does not contain control point manipulators because the control points can be controlled fully in the previewer (they do not have a *z* coordinate).

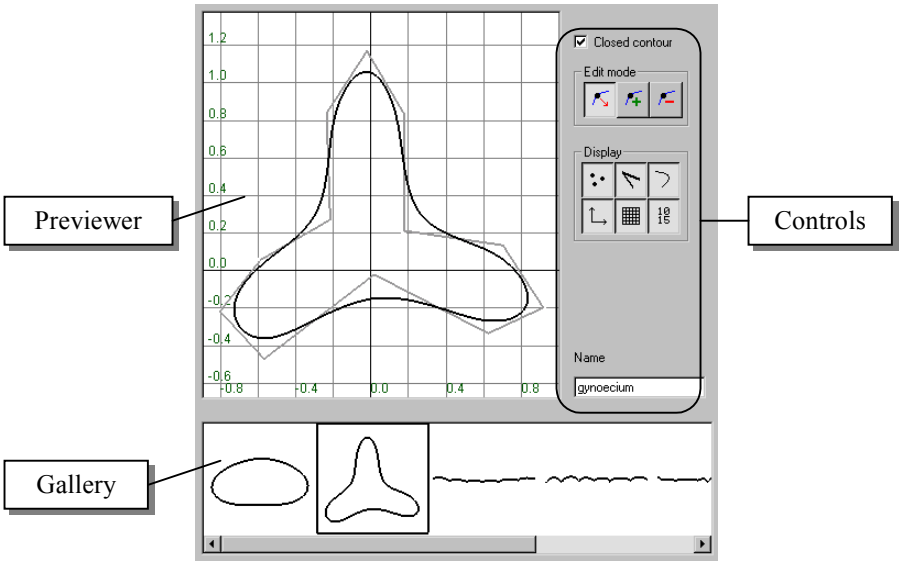


Figure 51 Screenshot of the contour editor

The functionality and responsibility of the components in an editor following the view /edit/gallery scheme can vary, depending on the type of elements being edited. In the material editor, elements of the gallery can be manipulated only using controls. In the case of the surface editor, some functionality available with controls is duplicated in the previewer. In the contour editor some functionality (manipulating control points) is available only via the previewer.

8.2. Continuous modeling mode

Recent improvements in the computational power of computers and the speed of graphics cards allowed revision of the process of working with models. The process introduced originally by Mercer in [Mer1991] that has been pursued so far involves the following cycle of actions:

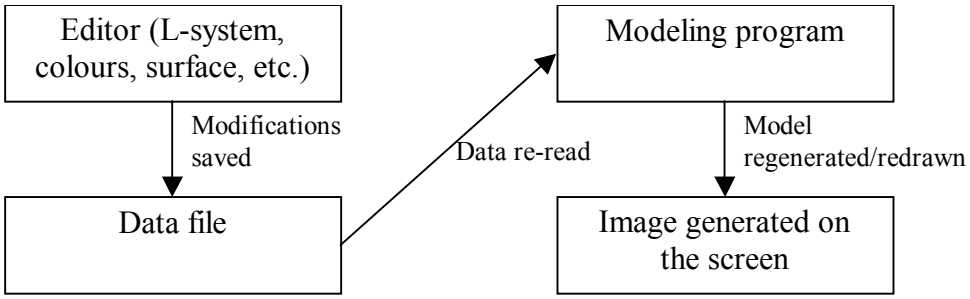


Figure 52 Edit-reread-regenerate scheme used when modeling

In this scheme the user first modifies a component of the model (L-system, colour specifications etc.), then saves it to a data file and makes the modeling program reread the modified information. The process is repeated until desired effect is achieved.

Continuous modeling mode frees the user from performing repetitive tasks of issuing saving and rereading commands, so that he/she can concentrate on the task at hand: *editing* of the data. After every change, the modifications are saved automatically and a request is sent to the modeling program to reread the modified data and redraw and/or regenerate the model. In this way, updated information transfers continuously from the editor to the modeling program. Continuous modeling mode has been added to the functionality of L-studio modeling environment.

8.3. Visually controlled parameters

The ability to manipulate parameters controlling the model is almost as important as the ability to express the model itself. Modification of the model code (L-system) qualitatively changes the model. On the other hand parameter modification changes quantitative features of the model. Modification of parameters is a way of performing experiments on the model by asking "what if?" questions. "What if the branching angle was different (see Figure 53)?" "What if asymmetry was different?" Etc.

Figure 53 Model of *Lychnis coronaria* (from [Pru1990]) generated for three different branching angles: 10°, 30° and 50°.

In the scope of my research I have improved the methods of controlling parameters by adding visual programming elements to the design of control panels. These improvements are discussed in more detail below.

Methods of controlling numerical parameters have been studied in the past [Mer1990, Mer1991]. Control panels cooperating with parameter editors introduced by Mercer [Mer1991] made it possible to control numerical and logical parameters visually. Panels contain controls such as sliders and buttons. Using these controls it is possible to manipulate parameters contained in text files without actually opening these files in a text editor.

The control panels are designed on a *per object* basis. In this way, the panels can always reflect the specific needs of every model or even different aspects of the same model, depending on the type of experiment or demonstration.

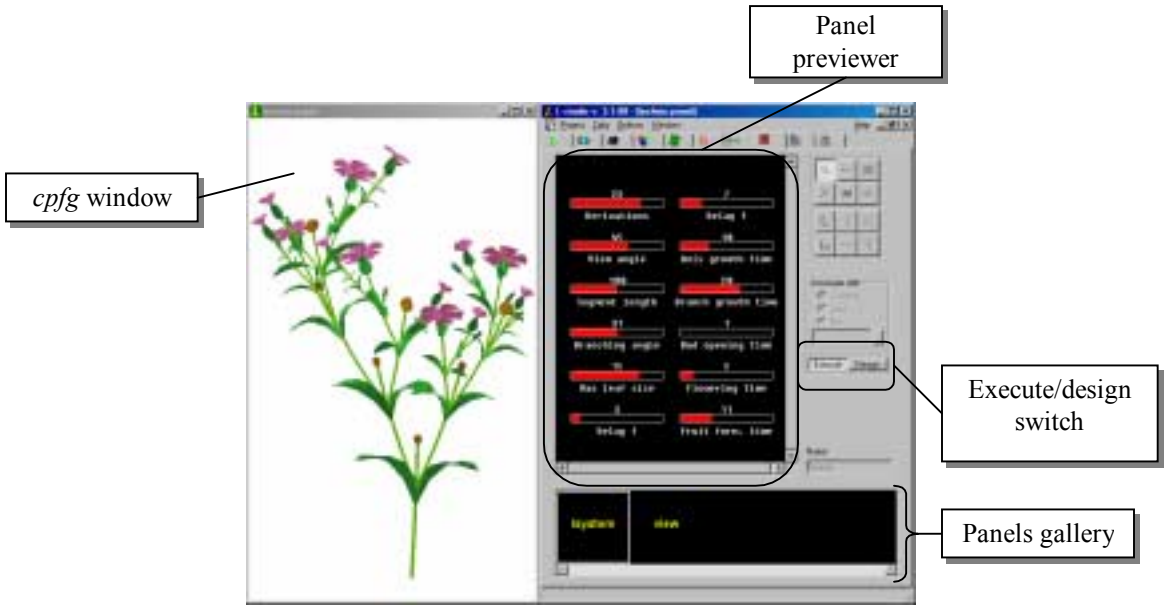


Figure 54 Model controlled by numerical parameters. The parameters are controlled by a panel.

In the original design [Mer1991] control panels had to be created by manually creating and editing text files (scripts) that describe the layout and functionality of the controls. The scripts are interpreted by *panel manager*. The panel manager is a program that reads the panel definition and then builds and visualizes the panel. The panel manager is also responsible for accepting the user input and translating it into the *parameter editor's* actions. The parameter editor is an external program that actually modifies the *data file*. In the original implementation, parameter editors were custom-written programs that internally callsed the ed and awk programs. Finally, the modified parameters are read by the application program (for example *cpfg*).

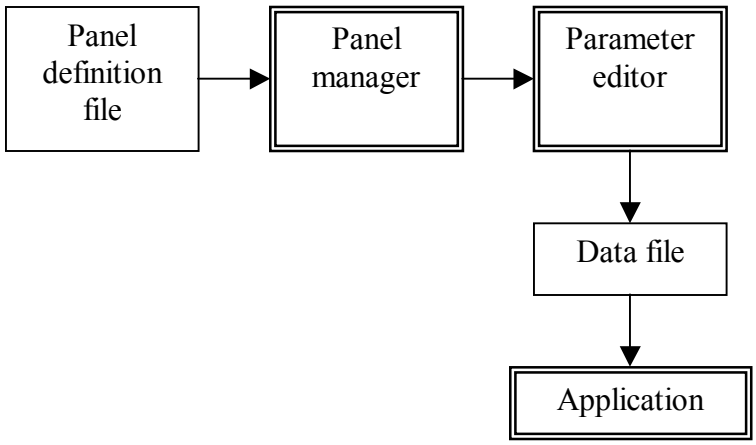


Figure 55 Communication flow involving the panel manager (after [Mer1991])

To simplify the building of control panels, I have researched the possibilities of applying interactive methods to this process. There are programs and utilities that allow interactive and visual design of elements of a GUI. Commonly known examples include *resource editors* in MS Visual Studio and Qt Designer. These tools make it possible to visually design elements of a GUI, such as menus or dialog boxes. Control panels are a special type of dialog box, which contain controls typical to dialog boxes: buttons, sliders, labels, group frames.

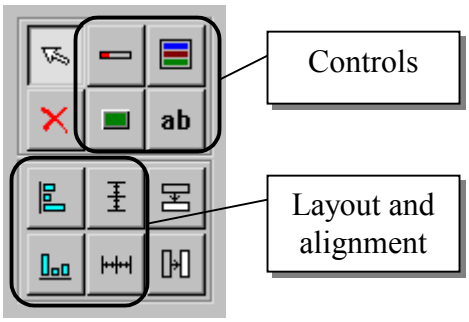


Figure 56 Visual design commands in the panel editor

This kind of tool has been implemented and incorporated into L-studio in a panel editor, which serves two functions. When in the *design mode* the panel editor allows the user to modify a panel:

- Add a control (slider, button, group box, label)
- Delete a control
- Move a control

- Arrange a group of controls (modify alignment, distribution)
- Specify and modify the actions associated with a control

In the panel editor the user also specifies the data file, which is controlled by the panel. In *execute mode*, the panel editor becomes the panel manager, building and displaying the controls. It also accepts and handles user input. Because the panel editor follows the view/edit/gallery design scheme, it is possible to handle more than one panel and perform the same operations on the set of panels that are performed in other galleries:

- Add new panels,
- Delete existing ones,
- Move panels between objects.

8.4. Visually defined functions

Productions in L+C can contain numerical expressions. The expressions may include arithmetic operators and function calls. In addition to the standard C++ mathematical functions and user-defined functions, they may also include graphically defined functions. Graphically defined functions are used in the following situations:

1) when it is difficult or impractical to find a formula that expresses the desired function, or
2) when it is necessary to be able to manipulate the values of the function locally.

In the process of modeling, some quantities defining the model cannot be described just by numbers. For example leaf length is not the same for all leaves even within the same plant or branch. Instead, it depends on the position of the leaf e.g. its placement on the stem. This parameter can be thought of as a function: it assigns exactly one value (leaf length) for every argument from the domain (position on the stem). Similarly, branching angle is also different for virtually every lateral branch. It can also be described as a function of position along the stem.

It is desirable to have access to this kind of parameters in the form of function calls from the L-system program, so that they can be used like any other function (trigonometric and other mathematical functions).

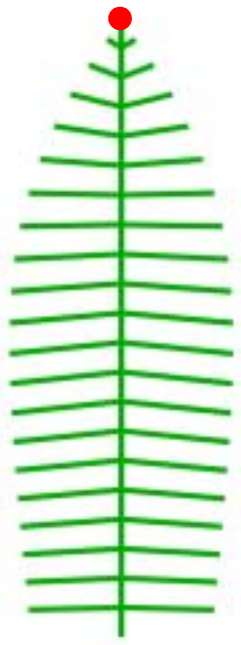
Listing 39 Model of a simple branching structure with lateral branches length and branching angle controlled by functions. Image generated by the L-system is on the right.

```
module A(int);
module L(float);

axiom: SetWidth(0.2) SetColor(2) A(1);

derivation length: 22;

A(n) :
{
  if (n<=20)
  {
    float arg = n/20.0;
    float brang = 90-90*Fangle(arg);
    produce F(1)
      SB Left(brang) F(Flength(arg)) EB
      SB Right(brang) F(Flength(arg)) EB
      A(n+1);
  }
  else
    produce F(1) SetColor(3) Circle(0.4);
}
```



The model in Listing 39 refers to two functions `Fangle` and `Flength`.

The information that defines these functions can come from different sources. Sometimes an algebraic formula is available. In that case, functions such as `Fangle` or `Flength` implement the formula and return the result. Sometimes the function can be defined by experimental data. In that case, the `Fangle` or `Flength` functions could calculate the results by reading the data from an array, an external file etc. However in some cases, only limited information is available about a function – e.g. the approximate shape of its plot. This idea can be found for example in [Lin1998, Lin1999]. Both functions (`Fangle` and `Flength`) used in the Listing 39 were defined by drawing the shape of the plot:

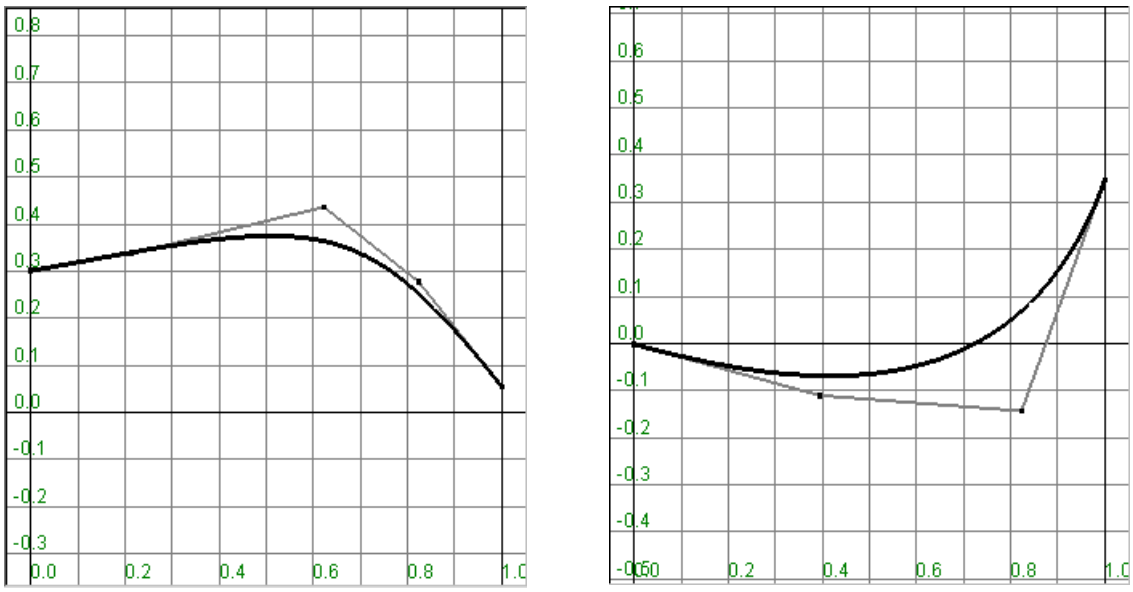


Figure 57 Functions used in the model in Listing 39

Rather than being specified by algebraic formulas, these functions are drawn using the *function editor*. The function editor is a b-spline curve editor with additional constrains, that make sure that the edited curve can be interpreted as a function *f(x) = y* for any $x \in [0,1]$. These constrains are:

- $p_1.x = 0$,
- $p_{last}.x = 1$,
- $p_n.x \leq p_{n+1}.x$.

Graphically defined functions can be easily modified by moving control points. For this reason, graphically defined functions represent extensions of visually controlled parameters.

Graphically defined functions do not replace functions specified by formulas or other algorithms that can be easily expressed in a general-purpose programming language such as C++. Doubtless it is much easier to type a formula such as `y = x*x` or `y = sin(x)`, rather than to draw the plot of these functions. Graphically defined functions create a new category of functional parameters that can be manipulated interactively by the user. The principal difference between analytic and graphically defined functions is that modifying a parameter in a formula (a coefficient in a polynomial or exponent in an exponential

function) usually changes the shape of the whole function. Graphically defined functions on the other hand make it possible to modify local properties of the plot, which are not easily obtainable when using algebraic formulas. Some results obtained by using visually defined functions have been described in [Pru2001].

Figure 58 shows the function editor and model of a fern controlled by graphically defined functions. Figure 59 shows examples of models generated using the graphically defined functions.

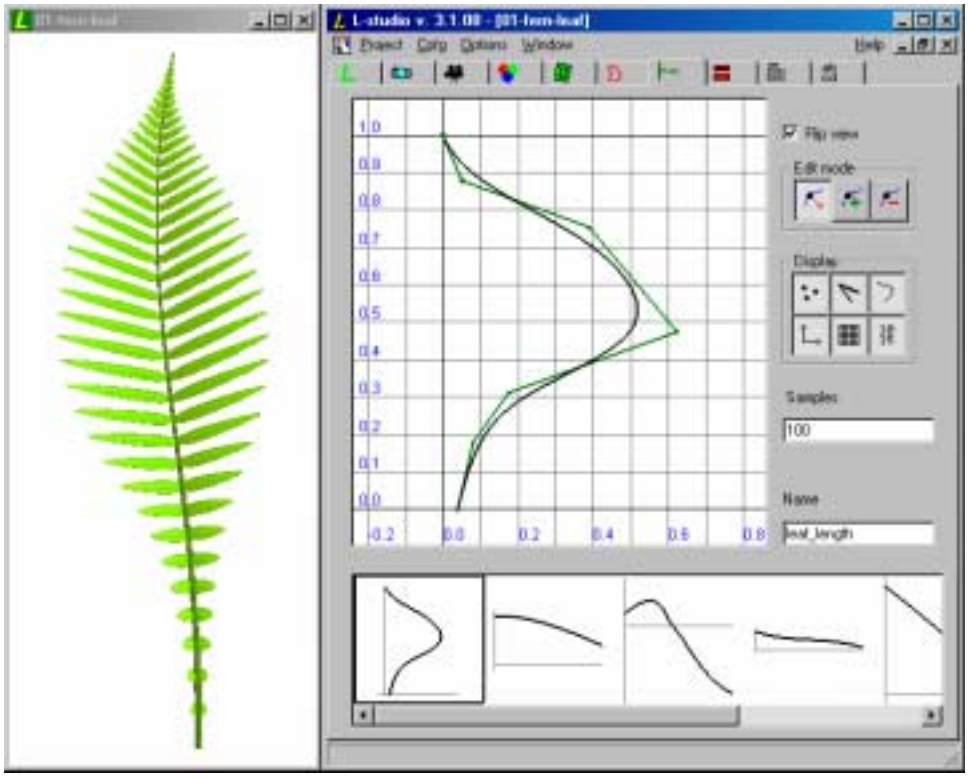


Figure 58 Model controlled by function parameters. The functions are controlled by the function editor

Figure 59 Models of *Pellaea falcata* and Indian paintbrush created using graphically defined functions (from [Pru2001])

8.5. Visual interaction with the model

The need for direct interaction with the model was recognized some time ago. Power et al. presented some research on visual manipulation of models generated using L-systems in [Pow1999]. In this paper the authors give an overview of their application *ilsa* (interactive L-string arranger) that allows a user to interactively manipulate the geometry of a plant model. The design of the *ilsa* is based on the flow of control presented in Figure 60.

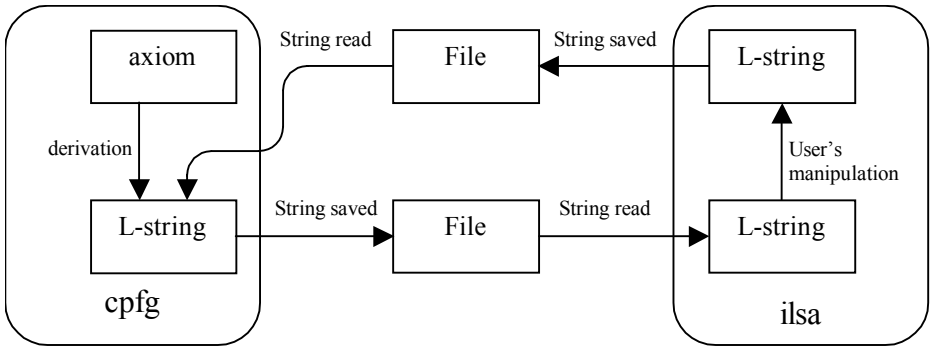


Figure 60 Information flow between cpfg and ilsa

To manipulate a plant model using cpfg/ilsa, the user first needs to create a model in cpfg. Once the model is generated the L-string is saved to a disk file. The file is then read by ilsa. Ilsa provides a display window where the user can interactively manipulate organs: turn, rotate and bend. Ilsa inserts corresponding turtle commands in the L-string to reflect the modifications. Then the string can be saved in a disk file and read back into cpfg. Work by Power provides a method for interactive manipulation of the geometry of generated structures.

I propose another method of interaction with models. My more general method is based on the ability to modify the string interactively by inserting a predefined module. The user can point to an element of the model on the screen and request to insert a predefined module (X) in the string. The module X is inserted in the string right before the module pointed to by the user. Figure 61 shows a visualization of the following L-system:

Listing 40 L-system implementing simple interactive pruning

```
Axiom: F1[+F2][-F3]
X → ;F4(0.2)%
```

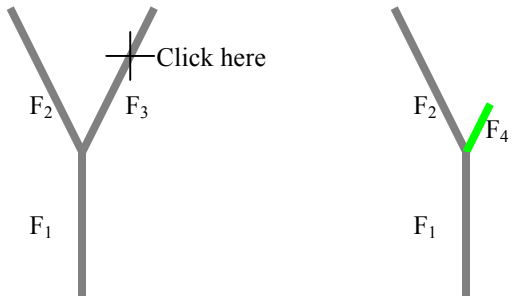


Figure 61 Module X inserted interactively

Initially the structure consists of three segments F_1, F_2 and F_3. When the user selects the branch corresponding to the module F_3 the string is modified:

$F_1[+F_2][-XF_3]$

During the next derivation step, the production p_1 is applied and the module X is replaced with the segment F_4, which is shorter and drawn using a different colour. The remainder of

the branch (module F_3) is removed by the module % (cut). In this way the user interactively modified the development of the model. This method can also be used to induce more complex actions than cutting a branch. For example, module X can be a signal that induces flowering in a selected branch. A short discussion on further applications is presented in 9.1.2.

9. Conclusions

9.1. Summary of contributions

The research described in this dissertation contributes to three areas in the domain of plant modeling:

a) L-systems as a mathematical formalism,
b) L-systems as a programming language,
c) Visual and interactive modeling.

The contributions belonging to the first two categories have been summarized and implemented in a new L-systems based modeling language L+C, which I have defined and implemented. Evaluation of the language from the conceptual and practical points of view is presented in section 9.1.1. Summary of my contributions in the domain of visual and interactive modeling are presented in section 9.1.2.

I have also created a plant modeling environment, L-studio. The environment is comprised of three principal elements: L-system based modeling programs (*cpfg* and *lpfg*) and a set of tools integrated into the program called L-studio. *Cpfg* is an L-system based plant modeling program. It was originally created by Prusinkiewicz. Further extensions were added by James, Hammel, Hanan, Měch and myself [Han1992, Mec1997a, Mec1998]. I have designed and implemented *lpfg*. *Lpfg* is an L-system based modeling program which implements and uses the L+C modeling language. The language is described in chapter 3.4. The user's manual for *lpfg* is presented in appendix A.

9.1.1. Evaluation of L+C

Users of L-systems have identified shortcomings of L-systems as a formalism and a modeling language. These shortcomings are described in chapter 3 together with proposed solutions. The number and significance of required modifications and extensions justified creation of a new language instead of extending the existing syntax. As a result I designed a new L-system-based modeling language L+C. It has been designed from scratch with a

well-defined set of requirements. This approach made it possible to include new elements in a cleanly and consistently.

The main conceptual contributions to the formalism of L-systems are fast information transfer and the new-context construct. Fast information transfer is an alternative method of propagating information and signals in models of both linear and branching structures. It has tow advantages over the traditional method of information transfer using context-sensitive productions:

a) It allows a clearer structure for models than in traditional L-systems, which need many derivation steps to propagate a signal.
b) The signal can be propagated throughout the entire structure in one derivation step. The time required is proportional to the number of modules in the string so the method is $O(N)$ as opposed to $O(N^2)$ when using context-sensitive methods.

New context facilitates implementation of fast information transfer in models without the use of a global variable (in the case of linear structures) or a stack (in the case of branching structures). Expressing fast information transfer using only local data (a modules' parameters and variables local to the productions) is consistent with the spirit of L-systems – expressing models in the local terms.

The introduction of user-defined types increased the expressive power of L-systems as a modeling language and allowed creation of models in which the modules are associated with many parameters. A large number of parameters associated with modules are required in (among others) genetic and biomechanical models.

An important decision in the design of L+C was to base its syntax on an existing general-purpose language, C++. L+C adds L-system-specific constructs to C++ while preserving all the expressive power of C++. The flow of control is governed by the declarative nature of L-systems rather than by the imperative paradigm as in C++.

This decision had two further effects:

a) The syntax of elements typical to C++ (e.g. declarations of structures, functions etc.) are the same as in C++. This diminishes the learning for new users of the language, if they are already familiar with C++.

b) The work required to implement the interpreter (or compiler) of the language has been reduced to a translator from L+C to C++.

Designing the translator was a valuable experience that showed what elements of the L+C constitute the essence of L-systems. In the process of designing it was necessary to identify elements of a program in L+C that must be translated and what additional information must be generated to link the L-system to the *generator* component of *lpfg*.

The improvements introduced by L+C come for a price. The simplest L-systems, which are very intuitive to grasp in the traditional notation, now look more complex. For example the L-system that generates the Koch curve (Listing 3 on page 8) when expressed in *cpfg* language reads:

```
Lsystem: 1
derivation length: 4
Axiom: F
F --> F+F--F+F
endlsystem
```

In L+C it becomes:

```
#include <lpfgall.h>
derivation length: 4;
axiom: F(1);
F(v) :
{
  produce
    F(1) Left(60) F(1) Right(120)
    F(1) Left(60) F(1);
}
```

The arrow notation of productions is replaced with a more complex one. The *cpfg* syntax was derived directly from the formal notation, which was intended to express simple concepts (for example `F` becomes `F+F--F+F`) in a simple way. It has been successfully extended to include parametric L-systems, L-systems with programming statements, environmentally sensitive L-systems and open L-systems.

In my opinion, the *cpfg* notation has reached the state when adding new elements to the syntax leads to obfuscated code, which is difficult to debug and maintain. Some examples have been presented already (see for example Listing 14 on page 36 and Listing 15 on page 37).

Also the decision to use single ASCII symbols or pair of symbols as identifiers of turtle commands in *cpfg* leads to expressions such as this one[11]:

Listing 41 Example of a complex successor written in *cpfg*

```
@OF(l)[,@v&(90)f(ran(0.01))^(90)@c(0.5*2)]
```

Because *cpfg* is not a free-form language, the possibilities of formatting productions are limited. I claim that the equivalent of code rewritten in L+C (see Listing 42), although longer, is more legible:

Listing 42 Equivalent of code from Listing 41 rewritten in L+C

```
Sphere0 F(1)
  SB
    DecColor RotToVert Down(90) f(ran(0.01)) Up(90) Circle(0.5*2)
  EB;
```

Better legibility is particularly evident in more complex models. Because L+C was designed to make it possible to write more complex models, it can be assumed that this goal has been achieved.

9.1.2. Visual and interactive aspects of modeling

I have also researched improvements in the domain of visual modeling. The concept of visually controlled parameters has been extended to include graphically defined functions. Corresponding visual tools to manipulate these functions have also been constructed and implemented. Graphically defined functions are one of the fundamental elements of *inverse modeling* [Pru2001]. The ability to manipulate the shape of the function's plot makes it is easy to experiment with models, for example by locally modifying the controlling functions.

The concept of continuous modeling leads to results that proved to be useful both in the course of modeling work and in the case of presentations and live demonstrations.

I have also implemented a new method of direct interaction with models. The result is the concept of modules that are interactively inserted into the model. Modules inserted

[11] An actual successor from a model of *Terminalia catappa* [Mec1996].

interactively do not break the link between the algorithm that generated the model, internal representation of the model and its graphical representation. The model can handle the modules inserted by the user and react on them.

Possible applications of interactively inserted modules include modeling of:

a) pruning,
b) grafting,
c) local mutations,
d) presence of pests,
e) application of pesticides.

9.2. Future work

In this section I briefly describe problems and questions that have been identified during my research but have not been addressed. These problems may constitute the foundation for future work on the modeling using L-systems and further development of L+C.

9.2.1. Missing elements

L+C does not include sub-L-systems. Sub-L-systems introduced by Hanan [Han1992] make it possible to create hierarchical models. Adding this concept to L+C is still an open problem because of differences of how local variables are handled in sub-L-systems and in C++.

Developmental surfaces [Mec1997a] are also not available in L+C. The problem of how to model growing surfaces such as leaves and petals is a very general and interesting challenge that definitely deserves work.

9.2.2. Problems worth revisiting

Object-oriented elements in L+C

One of the most interesting concepts that warrants further research of the L+C modeling language is the incorporation of concepts from object-oriented programming. In particular these concepts include:

a) inheritance.
b) Polymorphism.

c) Data hiding,

Inheritance

The concept of inheritance makes it possible to conceptually organize the elements of a model into a hierarchical structure. Elements that are lower in the structure inherit (share) features of the elements higher up. For example age can be considered a characteristic shared by all organs in a plant. It is therefore appropriate to define it at the top level of the hierarchy as a characteristic for all modules to share. Other examples that may characterize entire classes of modules, and therefore could be inherited (shared) include: amount of photosynthates produced by leaves, length and diameter of internodes, diameter of fruits.

In each case inheritance could be used to define modules that represent specific organs by adding new properties to the base modules.[12]

Polymorphism

In object-oriented languages polymorphism makes it possible to state that some calculations are to be performed, without giving details that may depend on the object type. Polymorphism is achieved using virtual methods (or functions), which have the same name but different meaning depending on the object type. For example, in the context of plant modeling, an important quantity is the amount of resources, such as carbon, allocated to specific modules. All organs use carbon for growth and maintenance, but functions that describe the amounts are different for different organs (for example, leaf vs. fruit). Thus the allocation of carbon may be defined using a virtual method at the level of organs and then specialized for individual organ types.

Data hiding

When dealing with data structures, manipulation can be performed using one of two approaches. The first approach is to access all members of the data structure directly. The values can be read and modified at will. The second approach is to encapsulate all operations that can be performed on the data structure into a set of functions. In the programming practice it has been found that this second approach works better. This is especially true when the members of a data structure are interdependent. The specialized functions guarantee that these interdependencies are always satisfied and the information

[12] A different concept of inheritance in L-systems has been proposed by Borovikov in [Bor1995]

represented by the structure is internally consistent. At present, the L+C language makes it possible to associate methods with data structures, but does not make it possible to associate methods with module types. Consequently, the concept of data hiding is not fully supported at the level of modules.

What are further consequences of deriving the string forward and backward?

More consideration can be given to the problems related to sequential derivation of the string and explicit control over the direction of the process. Fast information transfer relies on the ability to control the derivation direction. Also the `%` (cut) module introduced by Hanan [Han1992] requires the derivation to be performed from left to right. When the derivation is performed in the opposite direction branch cutting cannot be performed by skipping to the end of the current branch. What are other effects of sequential derivation forward or backward, and how the sequential execution of derivation steps alters the properties of L-systems? What is the relationship between fast information transfer and attribute grammars? More theoretical research would help answer these questions.

Dynamic data structures as modules' parameters

Currently it is clear how to use simple (C++ built-in) or compound types (structures) as modules' parameters. What is not clear is how to use non-trivial dynamic data structures (linked-lists, stacks etc.) as a modules' parameters. The main problem arises because the elements of these structures are allocated dynamically and require special handling when being deleted, copied etc. Various methods of resource transfer and resource management should be tested. In particular some consideration should be given to the use of STL-defined containers as a modules' parameters.

It is worth noting that the problems described above are a direct consequence of using C++ as the foundation of L+C. For example, these problems are not present in L-Lisp created by Erstad [Ers2002]. L-Lisp is an L-systems framework written in Common Lisp, a language with garbage collection mechanism.

Query modules vs. new context

Query modules contain meaningful information after the "interpretation step for the environment" has been performed. This step is performed after string rewriting is completed. In particular, query modules do *not* contain any meaningful information immediately after they are created (e.g. when they are in the new context). It would be interesting to consider the changes in the L+C language semantics to disallow query modules in the new context, or alternatively perform the interpretation while deriving. This could be applied only when deriving forward.

Module names

The L+C specifies that a module name must be a valid C++ identifier. This restriction disallows module names such as [and]. These identifiers have a well established position in L-systems notation and it is tempting to consider the possibility of allowing them as module identifiers.

However lifting the restriction on module names would imply a major redesign of the way the L+C to C++ translator works. Currently L2C works under the assumption that a single token is enough to recognize L+C constructs. But [and] symbols can appear in C++ code when they have different meaning (array element operator). Consequently allowing [and] as valid module identifiers would probably require full syntax analysis of the L+C source code.

9.3. Closing remarks

The main goal of my research was to improve the process of plant modeling. The L+C modeling language, implemented in lpfg, and the modeling environment L-studio are a practical realisation of the concepts I have introduced or extended in the scope of my research. The modeling system is distributed by the University of Calgary and is currently used in approximately 100 locations worldwide. L+C's applications are not limited to the plant modeling but also include applications in the domain of subdivision curves [Pru2002].

Appendices

A. *LPFG* user's guide

lpfg is a plant modeling program. The models are expressed using a formalism based on L-systems. The formalism, called the L+C modeling language adds L-systems specific constructs to the C++ programming language.

A.1. Hardware requirements

lpfg does not have any specific hardware requirements. It uses OpenGL to generate images it is therefore strongly recommended to use graphics cards capable of displaying graphics with resolution at least 1024x768 pixels at 24 or 32 bit depth. A mouse or an equivalent pointing device is also required.

A.2. Software requirements

lpfg runs under MS Windows operating systems (9x/Me/NT v. 4.0/2000). It requires a C++ compiler capable of generating Windows DLL's (Dynamic Link Libraries). *lpfg* was originally developed and tested using MS Visual C++ compiler v. 6.0.

A.3. Installation

lpfg is distributed together with L-studio. Refer to the L-studio installation guide on how to install it.

A.4. Command line options

lpfg is designed as an element of a modeling environment, such as L-studio or Vlab. Usually it will be invoked by the environment rather than directly by the user. This sections presents the command line switches supported by *lpfg*.

lpfg [-a] [-d] [-b] [-wnb] [-wnm] [-wr *w h*] [-wpr *x y*] [-wp *x y*] [-w *w h*] [–out *filename*] [*colormap_file*.map] [*material_file*.mat] [*animation_file*.a] [*functionset_file*.fset] [*drawparameters_file*.dr] [*viewparameters_file*.v] [*contourset_file*.cset] [*environmentfile*.e] *Lsystemfile*.l

`-a` – starts *lpfg* in the *animate mode*.

`-d` – starts *lpfg* in the *debug mode*.

`-b` – starts *lpfg* in the *batch mode*.

`-wnb` – no borders. The lpfg window is created without borders or title bar. Also the output console window is not shown. Used for demonstration purposes.

`-wnm` – no message window. The output console window is not shown.

`-wr` – specify relative window size. *w* and *h* parameters are numbers between 0 and 1 and specify the relative size of the lpfg window with respect to the screen.

`-wpr` – specify relative window position. *x* and *y* parameters specify the position of the top left corner relative to the top left corner of the screen.

`-wp` – *x* and *y* specify window's top left corner position in pixels relative to the top-left corner of the screen

`-w` – *w* and *h* specify window's size in pixels.

- Animate mode: *first frame* (as specified in the animation file) steps are performed, as opposed to `derivation length`.
- Debug mode: some information about the execution of the program is sent to the standard output. This mode is intended to be used by the developers of *lpfg*.
- Batch mode: no window is created. The simulation is performed and the final contents of the string is stored in the file specified. Only module names are stored in the file. This mode cannot be combined with the `-a` switch.

The only mandatory item is the L-system file. Command line parameters can appear in any order.

All the input file types are recognized based on their extension.

If no colormap file or material file is specified then default colormap is used.

A.5. User interface

A.5.1. View manipulation

- Rotation – *lpfg* uses XY rotation interface based on the *continuous XY rotation* as described by Chen et al. in [ref]. The model is rotated around the Y axis when the mouse is moved horizontally and around X axis when the mouse is moved vertically. To start rotating press left mouse button.
- Roll – to roll the model around the Z axis press Shift + middle mouse button. Moving the mouse to the right rotates the model clockwise, moving the mouse to the left rotates the model counter-clockwise.
- Zoom – to start press Ctrl + left mouse button or the middle mouse button. Moving mouse up zooms in, moving down zooms out.
- Pan – to start press Shift + left mouse button.
- Change frustum angle – to start press Ctrl + middle mouse button. Moving mouse up increases the angle, moving down decreases the angle. This operation has effect only in the perspective projection mode.

A.5.2. Menu commands

To display menu click the right mouse button inside the *lpfg* window.

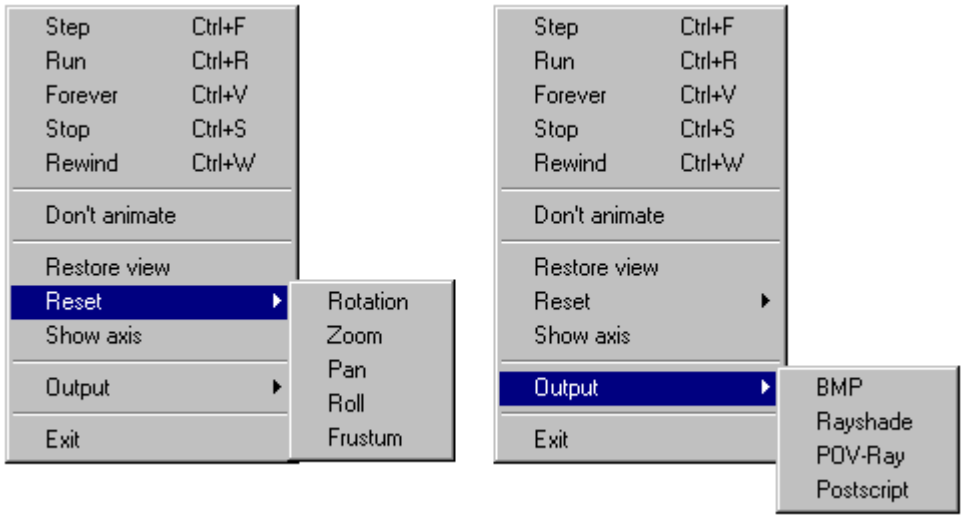


Figure 62 *Lpfg* menu

Step	Advances simulation to the next step. This may correspond to more than one derivation step if parameter *step* in the animate file is present and specifies a value greater than 1.
Run	Starts or resumes the animation.
Forever	Starts or resumes the animation. After the last frame is reached the animation returns to the *first frame* and continues.
Stop	Stops the animation.
Rewind	Resets the animation to the *first frame*.
Don't animate	Stops the animation and generates the image in the still mode (performs the number of derivation steps as specified in the `derivation length` statement).
Restore view	Resets rotation, zoom, pan, frustum and roll to the default values.
Reset → Rotation	Resets rotation.
Reset → Zoom	Resets zoom.
Reset → Pan	Resets pan.
Reset → Roll	Resets roll.
Reset → Frustum	Resets frustum (not implemented yet).
Show axis	Turns on or off display of coordinate system axis in the left lower corner.
Output → BMP	Creates image file *`filename`*`.bmp` containing the current state of the window. *Filename* is the name of the L-system file.
Output → Rayshade	Creates a rayshade file (not implemented yet).
Output → POV-Ray	Creates a POV-ray file (not implemented yet).
Output → Postscript	Creates a postscript file *`filename`*`.ps`. *Filename* is the name of the L-system file. All modules `F` are drawn as lines, even if `line style` is set to `cylinder`. If `line style` is `polygon` then modules F are drawn as lines of properly scaled width. The only other module supported is `Circle` and `Circle0`. No other modules are visualized.

A.6. L-system file

A typical L-system program file has the following format:

Listing 43 A typical L-system in L+C

```
#include <lpfgall.h>

derivation length: d;

// declarations of data structures

// declarations of functions

// module declarations

ignore: module list;

axiom: parametric word;

lcontext < predecessor > rcontext :
{
  …
  produce parametric word;
}
```

All elements of a program can appear in any order except for the following restrictions:

1) all elements referred to in a statement must be declared beforehand. For example: types used as parameters of a module must be declared before the module can be declared. Also modules that appear in the ignore or consider statement must be declared before.
2) Productions are matched in the order in which they are declared.

A.6.1. Mandatory elements

The mandatory elements in every L-system are the statements: `derivation length` and `axiom`.

A.6.2. Include files

The first line in the L-system is the include statement. The `lpfgall.h` include file includes the following standard header files:

- `memory.h` and `stdlib.h` – standard C header files. Required by the code generated by the L2C translator
- `lparams.h` – this file contains the declarations and definitions that are used by *lpfg*, L2C translator and the C++ code generated by the L2C translator. For example: maximum number of parameters per module, maximum number of modules in a production predecessor etc.
- `lintrfc.h` – this file contains declarations and definitions that are used by *lpfg* and the C++ code generated by the L2C translator. For example: types used for communication between the L-system and *lpfg*, predefined vector types (see section Predefined functions and structures)
- `lsys.h` – this file contains declarations and definitions required by the C++ code generated by the L2C translator. These include definitions of some predefined functions: `Forward()`, `Backward()`, etc.
- `stdmods.h` – this file contains declarations of predefined modules.

lpfg standard header files should be treated the same way as the standard C header files: they should never be changed or edited in any way. If they are models might not compile, stop working or *lpfg* may hang or crash.

In addition to the required include files any other include files can be also specified.

A.6.3. derivation length:

The `derivation length` statement has the following form:

`derivation length:` *integer expression* ;

The integer expression is any arithmetic expression that has value of type integer or a value that can be converted into an integer. The expression can be a function call (or it can contain a function call). The value of this expression is evaluated once when the model is generated in the *still mode*, but may be evaluated more than once in the *animate mode*. It is the user's responsibility to make sure that the expression has always the same value. For example, in the statement

```
derivation length: ++i;
```

the expression `++i` will return different value every time it is evaluated. In the animate mode this may result in an infinite number of steps.

A.6.4. Declarations of data structures and functions

Syntax of declarations of data structures and functions are the same as in C++.

A.6.5. Module declaration

Every module must be declared before it can be used in the axiom or in a production. The module declaration has the following form:

`module` *Identifier*(*parameters list*$_{opt}$); or
`module` *Identifier*;

The parameters list is a comma separated list of types. These are sample module declarations:

```
module A(int, float);
module B();
module C(data, float);
module D;
```

Module `A` is declared to have two parameters: one of type `int` and another one of type `float`. Module `B` is declared to have no parameters. Module `C` is declared to have two parameters of type `data` and `float` where data is a previously defined type (for example a structure). Module `D` is declared to have no parameters and uses the shortcut notation (without empty parentheses).

Note: All types must be declared before they can be used as parameters in module's declaration. In particular this applies to all user-defined types (structures).

Note: Parameters of modules (unlike parameters of functions) don't have names. Consequently it is illegal two give them names. For example:

```
module A(int id, float length);
```

will cause the L2C translator to signal an error. But it is legal and often useful to comment on module parameters:

```
module A(int /* id */, float /*length*/);
```

Note: Parameter type must be a *single* identifier. For example the following are valid C++ type specifiers but cannot be used as parameter types of modules:

```
unsigned int
char*
```

If a type of this kind is needed it is necessary to use `typedef` to create an identifier for that type. For example:

```
typedef unsigned int uint;
typedef char* text;
```

It is possible to declare a module and specify the module's numerical identifier in its declaration. The syntax for this declaration is:

`module` *Identifier*(*parameters list*$_{opt}$) = *constant expression*`;` or

`module` *Identifier* = *constant expression*`;`

This syntax should not be used in the user's L-systems and is reserved for the include file that declares standard modules (`stdmods.h`).

A.6.6. Axiom

The axiom statement has the following form:

`axiom:` *parametric word* ;

The parametric word is a sequence of modules. Initial value for every modules' parameters must be specified. Initial value of a parameter must be an expression of the same type as the type of the parameter. All conversion rules from C++ apply. In the following code:

```
module A(int, float);
```

```
axiom: A(5, 2.7-sin(M_PI/3)) A(1-abs(v), 5);
```

the axiom contains two modules of type `A`. Modules `A` have two parameters: an `int` and a `float`. The first module `A` will have its first parameter initialized with number 5. The second parameter will be initialized with the value of expression `2.7-sin(M_PI/3)` where `sin` is a previously declared function (in `math.h` in standard C++)

The second module `A` will have its first parameter initialized with value `1-abs(v)` where `abs` is a previously defined function (in `stdlib.h` in standard C++) and `v` is a variable previously defined. The second parameter will be initialized with a value of 5 (an integer) implicitly converted into a float `5.0f`.

If a module has no parameters its name can be followed by empty parentheses or the parentheses can be omitted altogether:

```
module B();
axiom: B() B;
```

In this example axiom contains two modules `B`. `B` has no parameters.

A.6.7. ignore, consider statements

ignore statement has the following form:

`ignore:` *list of modules* ;

The list of modules contains modules identifiers separated by blank characters (spaces, tabs or new lines). The list is terminated with a semicolon. For example:

```
ignore: A F P;
```

Syntax for the consider statement is analogous:

`consider:` *list of modules* ;

`ignore` and `consider` statements are mutually exclusive. By default all modules are considered when matching contexts. The `ignore` statement specifies the modules that should be ignored when matching contexts. The `consider` statement specifies the list of modules that should be considered when matching contexts. If `consider` statement is used then *only* the modules listed in the `consider` statement are considered. For detailed information on matching productions see section How the productions are matched.

Note: `SB` and `EB` modules are *always* considered. Listing them in the `consider` statement is unnecessary. Listing them in the `ignore` statement is allowed but has no effect.

A.6.8. Start, End, StartEach and EndEach control statements

The syntax for these statements is:

```
Start | End | StartEach | EndEach :
{
…
}
```

These statements should be treated as procedures. Any legal C++ statements are allowed inside their body. User can also specify here the direction of the derivation process (see predefined functions).

`Start` is always executed before axiom.

`StartEach` is executed before every derivation step. `EndEach` is executed after every derivation step.

`End` is executed after the last derivation step only when the program is not in the animate mode. This means:

- after the program has been started without the `-a` switch
- when the user selects **Don't animate** from the menu

A.6.9. Productions

There are three main types of productions:

a) context-free productions
b) context-sensitive productions
c) new-context-sensitive productions

Context-free productions have the following form:

strict predecessor :
{
…

}

Context-sensitive productions have the form

lcontext$_{opt}$ < *strict predecessor* > *rcontext*$_{opt}$:

{

…

}

At least one of contexts must be specified.

New-context sensitive productions have one of the following forms:

flcontext << *strict predecessor* > *rcontext*$_{opt}$:

{

…

}

lcontext$_{opt}$ < *strict predecessor* >> *frcontext*$_{opt}$:

{

…

}

Every component of the predecessor (contexts and the strict predecessor) consists of one or more modules. Modules in the predecessor must include names of their formal parameters. Formal modules are separated by commas. If a module has no parameters its name must be followed by a pair of parentheses `()`.

For example:

```
A(nl, fl) < B() > A(nr, fr) C(d, v) :
{ … }
```

specifies a production that has one module `B` as its strict predecessor. Module `A` is the left context and modules `A` and `C` are the right context. Names of the formal parameters must be unique within a predecessor. This is why parameters in the left context have postfix *l*. Production can contain any valid C++ code. Productions also usually contain one or more `produce` statements.

A.6.10. produce statement

Produce statement specifies production's successor. It is allowed only inside productions. It has the following form:

`produce` *parametric-word*$_{opt}$;

Parametric-word has the same syntax as in `axiom`. If it is omitted then the successor of the production is empty. Effectively it removes the modules present in the strict predecessor from the derived string.

Examples:

```
produce A(4, 5.2) B A(1, 0.5);
produce ;
```

If a production does not execute a produce statement then *lpfg* will continue searching for other productions matching the current position in the string. If no production is found then default (identity) production is applied.

A.6.11. Decomposition rules

Decomposition rules make it possible to decompose a module in the string into several components. After axiom and every derivation step a *decomposition step* is performed. Decomposition is performed as long as the string does not contain any modules that can be further decomposed or the *maximum decomposition depth* is reached. Syntactically decomposition rules are very similar to regular productions except for the following differences:

- only one module in the strict predecessor is allowed,
- decomposition rules are always context-free.

When the statement `decomposition:` is present in the L-system it specifies that all the following rules are decomposition rules until the end of the source file or until `production:` or `interpretation:` statement is encountered. To specify maximum decomposition depth the `maximum depth:` statement is used. It must be placed in the global scope after the `decomposition:`. The syntax of the maximum depth statement is as follows:

`maximum depth:` *integer-expression* ;

The default maximum decomposition depth is 1.

Note: L-system can contain many decomposition sections but only one `maximum depth:` statement is allowed for decomposition. The same applies to the *interpretation rules*.

Decomposition rules can be *recursive*, e.g. the module in the strict predecessor can appear in the successor. For example:

```
#include <lpfgall.h>
module A(int);
axiom: A(5);
derivation length: 0;
decomposition:
maximum depth: 6;
A(n) :
{
  if (n>0)
    produce F(1) A(n-1);
}
```

The L-system above will generate five modules `F(1)`.

Note: decomposition is internally implemented by a recursive call to a function. If the maximum depth is a very large number the thread stack might overflow causing *lpfg* to crash.

A.6.12. Interpretation rules

Interpretation rules are syntactically very similar to the decomposition rules. To specify interpretation rules the `interpretation:` statement must be specified. Like the decomposition rules interpretation rules must have exactly one module in the strict predecessor and must be context-free. At most one `maximum depth:` statement is allowed per L-system.

Interpretation rules are executed only during the interpretation of the string. Modules produced by interpretation rules are not inserted into the string but are just used as commands for the turtle during the interpretation steps.

The interpretation step is performed in the following cases:

1. When redrawing the model in the window
2. When generating output file (rayshade, POVray, postscript)
3. When calculating the *view volume*.
4. After axiom and each derivation step if any of the productions contains query or communication modules

Note: interpretation and decomposition rules can be very helpful in properly expressing models. They can be used to separate the functional aspect of the model from its visual aspect. But these rules can be also misused. In particular interpretation rules might add time overhead. It is a matter of experience and good design intuition to use them wisely and effectively.

Note: in *lpfg* it is possible to specify regular productions after decomposition and interpretation rules. To specify regular productions use the `production:` statement. This possibility leads to another methodology of writing models. Instead of dividing the model into production, decomposition and interpretation sections all rules that apply to one type of module can be grouped together. For example:

```
production:
A() : { … }
decomposition:
A() : { … }
interpretation:
A() : { … }

production:
B() > A() : { … }
decomposition:
B() : { … }
```

Etc.

A.6.13. Predefined functions and structures

Here is the summary of the predefined functions and structures provided by *lpfg*:

`void Forward()`

This function specifies that the derivation of the string should be performed forward – from left to right. This is default.

`void Backward()`

This function specified that the derivation of the string should be performed backward – from right to left.

`Forward` and `Backward` can be used anywhere in the code where it is legal to call a function. They take effect the next time derivation is performed. In particular if called in the `StartEach` statement they affect the current derivation step.

`void Printf(const char*, …).`

This function is similar to the standard C function `printf`. Its use is recommended over the `printf` for the following reasons:

- Output generated by `printf` is not stored in the `lpfg.log` file.
- In the future releases *lpfg* might be not connected to any console but instead provide its own output window (like *cpfg*'s message log). In that case output of `printf` would not be visible anywhere.

`float ran(float range)`

Generates a pseudorandom number uniformly distributed in the range [0, range).

`float func(int id, float x)`

This function returns the value of the function specified in the function-set file (if one is present in the command line). First parameter specifies the ordinal number of the function as in the `.fset` file. It must be in the range [1, *num of functions*]. The second is the *x* value for which the *y* value is requested. Parameter *x* must be in the range [0, 1].

If the parameter id is incorrect (outside the range) value 0 is returned and warning message is printed. If the parameter x has invalid value then:

- if `x<0` then `func(id, 0)` is returned
- if `x>1` then `func(id, 1)` is returned

In addition *lpfg* provides four structures that represent vectors. The structures are:

```
struct V2f
{ float x, y; };
struct V3f
{ float x, y, z; };
struct V2d
{ double x, y; };
struct V3d
{ double x, y, z; };
```

These structures are used as parameters for some predefined modules. They can also be used in the user's code in the L-system. Additionally if the preprocessor's symbol `NOAUTOOVERLOAD` is not defined before `#include <lpfgall.h>` these structures receive additional functionality: operators for addition of two structures of the same type and multiplying a vector by a number. For example:

```
V2f a(1.5, 2.0), b(0, 0.5);
V2f c = a + 2.5*b;
```

Refer to the file `lintrfc.h` in the `lpfg/include` directory to see full definition of these structures.

A.6.14. Predefined modules

The following table lists all the predefined modules.

Module	Description	Equivalent in *cpfg*

Modeling branching structures

`SB()`	Starts a new branch by pushing the current state of the turtle on the turtle stack.	`[`
`EB()`	Ends a branch by popping the state of the turtle from the turtle stack.	`]`
`Cut()`	Cuts the remainder of the current branch. When detected in the string during the generation process, the module and all following modules up to the closest unmatched `EB` module are ignored for derivation purposes. If no unmatched `EB` module can be found, symbols are ignored until the end of the string.	`%`

Changing position and drawing

Turtle commands

`F(float /*d*/)`	Moves forward a step of length `d` and draws a line segment from the original position to the new position of the turtle.	`F(d)`
`f(float /*d*/)`	Moves forward a step of length `d`. No line is drawn.	`f(d)`
`MoveTo (float /*x*/, float /*y*/, float /*z*)`	Sets the turtle position to `x`, `y`, `z`.	`@M(x,y,z)`
`MoveTo2f (V2f /*p*/)`	Moves the turtle to point `p`. *z* coordinate is assumed to be 0.	`@M`
`MoveTo2d (V2d /*p*/)`	Same as `MoveTo2f`	`@M`
`MoveTo3f (V3f /*p*/)`	Same as `MoveTo2f` except that the *z* coordinate is specified by the parameter `p`.	`@M`
`MoveTo3d (V3d /*p*/)`	Same as `MoveTo3f`	`@M`

Affine geometry support

`Line2f (V2f /*p1*/, V2f /*p2*/)`	Draws a line from the point specified by `p1` to the point `p2`. *z* coordinates are assumed to be 0. After the interpretation of the module the turtle position is equal to `p2`. Heading, left	

	and up vectors are not changed. If the distance between p1 and p2 is less than ε[13] the module is ignored.	
`Line2d (V2d /*p1*/, V2d /*p2*/)`	Same as `Line2f`	
`Line3f (V3f /*p1*/, V3f /*p2*/)`	Same as `Line2f`, except that z coordinates are specified in the `p1` and `p2` parameters	
`Line3d (V3d /*p1*/, V3d /*p2*/)`	Same as `Line3f`	
`LineTo2f (V2f /*p*/)`	Draws a line from the current turtle position to the point `p`. *z* coordinate is assumed to be 0. After the interpretation of the module the turtle position is equal to `p`. Heading, left and up vectors are not changed. If the distance from the current position to `p` is less than ε the module is ignored.	
`LineTo2d (V2d /*p*/)`	Same as `LineTo2f`	
`LineTo3f (V3f /*p*/)`	Same as `LineTo2f`, except that *z* coordinate is specified by the parameter `p`.	
`LineTo3d (V3d /*p*/)`	Same as `LineTo3f`	
`LineRel2f (V2f /*p*/)`	Draws a line from the current turtle position to the point `p2` = (*turtle position*) + `p`. *z* coordinate is assumed to be 0. After the interpretation of the module the turtle position is equal to `p2`. Heading, left and up vectors are not changed. If the length of vector `p` is less than ε the module is ignored.	
`LineRel2d (V2d /*p*/)`	Same as `LineRel2f`	
`LineRel3f (V3f /*p*/)`	Same as `LineRel2f`, except that *z* coordinate is specified by the parameter `p`.	
`LineRel3d (V3d /*p*/)`	Same as `LineRel3f`	

Turtle rotations

`Left (float /*a*/)`	Turns left by angle `a` around the **U** axis	`+(a)`
`Right (float /*a*/)`	Turns right by angle `a` around the **U** axis	`-(a)`
`Up(float /*a*/)`	Pitches up by angle `a` around the **L** axis	`^(a)`

[13] ε is defined as 0.00001

`Down (float /*a*/)`	Pitches down by angle `a` around the **L** axis	`&(a)`
`RollL (float /*a*/)`	Rolls left by angle `a` around the **H** axis	`\(a)`
`RollR (float /*a*/)`	Rolls right by angle `a` around the **H** axis	`/(a)`
`TurnAround()`	Turns around 180 degrees around the **U** axis. This is equivalent to `Left(180)` or `Right(180)`. It does not roll or pitch the turtle	`|`
`SetHead (float /*hx*/, float /*hy*/, float /*xz*, float /*ux*/, float /*uy*/, float /*uz*)`	Sets the heading vector of the turtle to `hx`, `hy`, `hz` and the up vector to `ux`, `uy`, `uz`. The values do not need to specify normalized vectors. The module is ignored if any of the following is true: a) `hx`,`hy`,`hz` specify a vector of length less than ε b) `ux`,`uy`,`uz` specify a vector of length less than ε c) Length of the cross product of new **H** and **U** is less than ε	`@R (hx,hy,hz, ux,uy,uz)`
`RollToVert()`	Rolls the turtle around the **H** axis so that **H** and **U** line in a common vertical plane with **U** closest to up.	`@v`

Changing turtle parameters

`IncColor()`	Increases the value of the current colour index or material index by one	`;`
`DecColor()`	Decreases the value of the current colour index or material index by one	`,`
`SetColor (int /*n*/)`	Sets the value of the current colour index or material index to `n`. If `n` is less than 1 or greater than 255 the module is ignored.	`;(n)` `,(n)`
`SetWidth (float /*v*/)`	Sets the value of the current line width to `v`. If `v`≤0 the module is ignored.	`#(n)` `!(n)`

Drawing circles and spheres

`Circle (float /*r*/)`	Draws a circle of radius `r` at the current turtle position in the XY plane.	`@o(d)` `@c(d)`
`Sphere (float /*r*/)`	Draws a sphere of radius `r` at the current turtle position.	`@O(d)`
`Sphere0()`	Draws a sphere of diameter equal to the current line width	`@O`
`Circle0()`	Draws a circle of diameter equal to the current line width	`@o`

		`@c`

Drawing bicubic parametric surfaces

`Surface (int /*id*/, float /*scale*/)`	Draws the predefined surface identified by the identifier `id` at the current location and orientation. The surface is scaled by the factor `scale`. Surfaces are specified in the view file. The first surface specified in the view file has `id`=0.	`~`

Drawing generalized cylinders

`CurrentContour (int /*id*/)`	Sets the contour specified by `id` as the current contour for generalized cylinders. If `id` equal to 0 is specified then the default contour (circle) is used.	`@#(id)`
`StartGC()`	Starts a generalized cylinder in the current turtle position. (Functionality not fully implemented yet)	`@Gs`
`PointGC (int /*n*/)`	Specifies a control point on the central line of the generalized cylinder. The value `n` specifies how many mesh strips are drawn between the control point and the previous one. (Functionality not fully implemented yet)	`@Gc(n)`
`EndGC (int /*n*/)`	Ends a generalized cylinder. The parameter `n` specifies the number of strips as for the module `PointGC`. (Functionality not fully implemented yet)	`@Ge`

Drawing mesh

`MeshPoint()`	Specifies a mesh point.	

Tropism

`SetElasticity (int /*id*/, float /*v*/)`	Sets the elasticity parameter of tropism `id` to value `v`.	`@Ts`
`IncElasticity (int /*id*/)`	Increments the elasticity parameter of tropism `id` by the elasticity step parameter of the tropism.	`@Ti`
`DecElasticity (int /*id*/)`	Decrements the elasticity parameter of tropism `id` by the elasticity step parameter of the tropism	`@Td`

Query and communication modules

`GetPos (float /*x*/, float /*y*/, float /*z*/)`	Queries the current turtle position. If any *query module* is present in the predecessor of any production in the L-system a special interpretation step is performed after each generate step, when productions are applied. The string is interpreted even if no drawing occurs. During the interpretation the three parameters of the module are set to the `x`, `y` and `z` coordinates of the current turtle position.	`?P(x,y,z)`
`GetHead (float /*x*/, float /*y*/, float /*z*)`	Queries the current turtle heading vector.	`?H(x,y,z)`
`GetLeft (float /*x*/, float /*y*/, float /*z*)`	Queries the current turtle left vector.	`?L(x,y,z)`
`GetUp (float /*x*/, float /*y*/, float /*z*)`	Queries the current turtle up vector.	`?U(x,y,z)`
`En(float …)`	Communication modules used to send and receive environmental information.	`?E(v)`

Miscellaneous

`Label(Text`[14] `str)`	Prints the string `str` in the drawing window at the current turtle location.	`@L(str)`

A.7. Other input files

A.7.1. Animation parameters file

Command	Comments
`first frame:` *n*	Derivation step to be interpreted as the first frame. Default is 0. **Note:** in *cpfg* default first frame is 1. This is why Rewind in *cpfg* rewinds to the first derivation step, while in *lpfg* Rewind rewinds to axiom.

[14] `Text` is `typedef`'ed as `const char*` in `lintrfc.h`

`last frame: n`	Derivation step to be interpreted as the last frame. Default is the number of derivation steps.
`swap interval: t`	Time interval between frames. (Currently not implemented)
`step: n`	Number of derivation steps between drawing of frames. Default is 1
`Double buffer: on|off`	Specifies if the double buffer mode should be used. Default is `on`.
`clear between frames: on|off`	Specifies whether to clear the screen between frames. Default is `on`.

A.7.2. Draw/view parameters file

Drawing and viewing parameters are stored in the view file. This file can have extension `.v` or `.dr`. View file is preprocessed by standard C++ preprocessor therefore use of comments (both C style `/* … */` and C++ style `//`) as well as `#define`'s `#if`'s and all other standard preprocessor directives are allowed. The commands are interpreted in the order in which they appear in the file. If there two or more commands that specify the same parameter the last one takes precedence. This does not apply to commands that specify new set of parameters every time they appear (e.g. *lights*, *tropisms*). Every command must be contained in a single line.

Command	**Comments**
Setting the view	
`projection: parallel|perspective`	Default is `parallel`.
`scale: s`	*s* specifies the size of the final image on the screen. 1.0 corresponds to full size. Default is 1.1.
`min zoom: v`	*v* specifies the minimum value of zooming factor (see Interactive view manipulation). Default is 0.05.
`max zoom: v`	*v* specifies the maximum value of zooming factor (see Interactive view manipulation). Default is 50.
`line style: style`	*Style* must be one of the following: `pixel`, `polygon` or `cylinder`. Default is `pixel`.
`front distance: x`	*x* specifies the front clipping plane
`back distance: x`	*x* specifies the back clipping plane
Rendering parameters	

`z buffer: on|off`	Default is `off`.
`render mode:`	*Mode* must be one of the following: `filled`, `wireframe` or `shaded`. Default is `filled`.
`light:` *command*$_1$ *command*$_2$ …	*Command* must be one of the following: `O: x y z` origin of point light source `V: x y z` vector of directional source `A: r g b` ambient `D: r g b` ambient `S: r g b` specular `P: x y z e c` spotlight with the direction (`x`,`y`,`z`), exponent `e`, cutoff angle `c` `T: c l q` attenuation factors. Up to 8 lights can be specified. 8 is the minimum number of lights that must be supported according to the OpenGL specifications

Other commands

`surface:` *filename txid*$_{opt}$	*Filename* is the filename of a surface (`.s`) file. *txid* if present specifies identifier of the texture associated with the surface. See description of the module `Surface` in Predefined modules. **Note:** this command can be dropped in a future version when the gallery of surfaces is introduced.
`texture:` *filename*	*Filename* specifies the image file that contains the texture. Both width and height of the image must be powers of 2. Textures are indexed starting at 0. Currently only RGB files are supported.
`tropism:` *command*$_1$ …	*Command* must be one of the following: `T: x y z` tropism vector (required) `A: a` angle. Default is 0. `I: x` intensity. Default is 1 `E: e` elasticity. Default is 0 `S: de` elasticity step. Default is 0. Any number of tropisms can be specified in the view file.
`torque:` *command*$_1$ …	*Command* must be one of the commands valid for tropism except for `A`.

A.7.3. Environment parameters file

Environment parameters file has extension `.e`.

Command	Remarks
`executable:` *`command`*	Specifies the executable of the environmental process together with its optional command line parameters
`communication type: pipes|sockets|memory|files`	Ignored. The only communication supported in the current version is files.
`following modules: on|off`	Ignored. No following modules are sent to the environment.
`turtle position:` *`format`* `turtle heading:` *`format`* `turtle left:` *`format`* `turtle up:` *`format`* `turtle line width:` *`format`* `turtle scale factor:` *`format`*	C-like format string used when sending turtle parameters. All are optional but most environmental programs will require at least turtle position. For example: `turtle position: P: %f %f %f`
`verbose: on|off`	Verbose mode generates additional information about the details of the communication

A.7.4. Miscellaneous input files

A.7.4.1. Colourmap file

Specifies 256 colours. Colourmap mode is used to create schematic images. See material file.

For the description of the file format see the document L-studio, Cpfg, Lpfg – format description.

A.7.4.2. Material file

Specifies 256 materials. Materials are specified by the following components: ambient, diffuse, specular, emission and transparency. See OpenGL documentation for further explanation. Material mode is used to create realistic images.

For the description of the file format see the document L-studio, Cpfg, Lpfg – format description.

A.7.4.3. Surface file

Specifies surfaces composed of one or more Bézier patches.

For the description of the file format see the document L-studio, Cpfg, Lpfg – format description.

A.7.4.4. Function-set file

Specifies functions of one variable. The functions are defined as B-spline curves constrained in such a way that they assign exactly one y to every x in the normalized function domain [0, 1].

For the description of the file format see the document L-studio, Cpfg, Lpfg – format description.

A.7.4.5. Contour-set file

Specifies contours defined as planar B-spline curves. The curves are considered as cross-sections of generalized cylinders.

For the description of the file format see the document L-studio, Cpfg, Lpfg – format description.

A.7.4.6. Textures

Currently the only supported format of textures is RGB. Textures in the RGB format may contain Alpha (transparency) channel.

B. Listings

B.1. Iterating L-system string in the traditional representation

Listing 44 Function FindNextModule – traditional string representation

```
// the function returns the pointer to the next module
// or NULL if end of string is found
const char* FindNextModule(const char* pCP)
{
  // Input: pCP – current position in the string
  pCP++;
  if (0 == *pCP) // end of string found
    return NULL;
  else if ('(' != pCP) // current module has no parameters
    return pCP;
  else // current module has parameters
  {
    do
    {
      pCP += sizeof(float); // skip the parameter
      pCP++; // skip to the coma or right parenthesis
    }
    while (',' == *pCP);
    assert(')' == *pCP); // otherwise the string is corrupted
    pCP++;
    if (0 == *pCP)
      return NULL; // end of string found
    else
      return pCP;
  }
}
```

Listing 45 Function FindPreviousModule – traditional string representation

```
// the function returns the pointer to the previous module
// it assumes that the current module is not the first module
const char* FindPreviousModule(const char* pCP)
{
  // Input: pCP – current position in the string
  // pBOS is the pointer to the beginning of the string
  assert(pCP>pBOS); // don't call me if this is the first module
  pCP--;
  if (pCP != ')')
    return pCP;
  else
  {
    do
    {
```

```
      pCP -= sizeof(float); // skip the parameter
      pCP--; // skip to the coma or left parenthesis
    }
    while (',' == *pCP);
    assert('(' == *pCP); // otherwise the string is corrupted
    pCP--;
    return pCP;
  }
}
```

B.2. Iterating L-system string in the new representation

Listing 46 Function FindNextModule – new string representation

```
const char* FindNextModule(const char* pCP)
{
  // Input: pCP – current position in the string
  short int ModuleId;
  // retrieve the current module id
  memcpy(&ModuleId, pCP, sizeof(short int));
  // get the size of its parameters
  int SizeOfParameters = GetParametersSize(ModuleId);
  if (SizeOfParameters==0) // no parameters
  {
    pCP += sizeof(short int); // just skip the module id
    return pCP;
  }
  else
  {
    pCP += sizeof(short int); // skip the module id
    pCP += SizeOfParameters;  // skip the parameters
    pCP += sizeof(short int); // skip the trailing id
    return pCP;
  }
}
```

Listing 47 Function FindPreviousModule – new string representation

```
const char* FindPreviousModule(const char* pCP)
{
  assert(pCP>pBOS); // don't call me if this is the first module
  // skip to the previous module id
  pCP -= sizeof(short int);
  short int ModuleId;
  // retrieve the previous module id
  memcpy(&ModuleId, pCP, sizeof(short int));
  // get the size of its parameters
  int SizeOfParameters = GetParametersSize(ModuleId);
  if (SizeOfParameters==0)
   return pCP; // no parameters – we're done
  else
  {
    pCP -= SizeOfParameters;  // skip the parameters
    pCP -= sizeof(short int); // skip the module id
```

```
    return pCP;
  }
}
```

B.3. Predefined types provided by lpfg in the file lintrfc.h

```
template <typename f>
struct V2
{
  V2() : x(0.0f), y(0.0f) {}
  V2(f nx, f ny) : x(nx), y(ny) {}
  V2<f> operator+(V2<f> l, V2<f> r)
  { return V2<f>(l.x+r.x, l.y+r.y); }

  // Multiplication of vector by a scalar
  V2<f> operator*(f r, V2<f> l)
  { return V2<f>(l.x*r, l.y*r); }

  // Scalar product
  f operator*(V2<f> l)
  { return x*l.x + y*l.y; }

  f x, y;
};
typedef V2<float> V2f;
typedef V2<double> V2d;


template <typename f>
struct V3
{
  V3() : x(0.0f), y(0.0f), z(0.0f) {}
  V3(f nx, f ny, f nz) : x(nx), y(ny), z(nz) {}
  V3<f> operator+(V3<f> l, V3<f> r)
  { return V3<f>(l.x+r.x, l.y+r.y, l.z+r.z); }
  V3<f> operator-(V3<f> l, V3<f> r)
  { return V3<f>(l.x-r.x, l.y-r.y, l.z-r.z); }

  // Multiplication of vector by a scalar
  V3<f> operator*(f r, V3<f> l)
  { return V3<f>(l.x*r, l.y*r, l.z*r); }

  // Scalar product
  f operator*(V3<f> r, V3<f> l)
  { return r.x*l.x + r.y*l.y + r.z*l.z; }

  // Vector product
  V3<f> operator%(V3<f> r, V3<f> l)
  { return V3<f>(
      r.y*l.z-r.z*l.y,
      r.z*l.x-r.x*l.z,
      r.x*l.y-r.y*l.x);
  }
  V3<f> operator+=(V3<f> l)
  {
```

```
    x += l.x; y += l.y; z += l.z;
    return *this;
  }
  f x, y, z;
};

typedef V3<float> V3f;
typedef V3<double> V3d;
```

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