CHAPTER 3

THE L+C PLANT-MODELLING LANGUAGE

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Abstract. L+C is a modelling language that combines features of L-systems and C++. It extends the L-system formalism with the notion of fast transfer of information, and supports a number of standard programming constructs absent from its predecessor, the cpfg language. These include modules with structured parameters, productions with multiple successors, and user-definable functions. Visualizations of L-system models can be enhanced using multiple views and the selective display of frames. These features extend the overall range of simulation models that can be conveniently expressed using L-systems, and are particularly advantageous when creating and visualizing complex plant models. A biomechanical model of a growing pendulous branch is given as the key example.

INTRODUCTION

L-studio (Windows) and the Virtual Laboratory (Linux and Mac) are related plant-modelling packages distributed by the University of Calgary, Canada. Each of the L-studio and Vlab systems consists of: (a) two L-system-based simulation programs, cpfg and lpfg; (b) a modelling environment that provides auxiliary modelling tools and a graphical interface for creating and manipulating models; (c) a library of programs for simulating environmental processes that affect plant development; (d) a set of sample models; and (e) a graphical browser for organizing and accessing models on both local and remote machines (Prusinkiewicz 2004).

The simulation programs cpfg and lpfg are at the heart of both L-studio and Vlab. Their design has been guided by two key objectives: (a) the programs should be suitable for modelling and simulating a wide range of structures and developmental processes in plants, and (b) the programs should support diverse visualization techniques, from schematic to realistic. These objectives are addressed by allowing users to specify models in specialized programming languages, which are based on the formalism of L-systems (Lindenmayer 1968a; 1968b; 1971). The modelling language for the cpfg simulator was developed first (Prusinkiewicz and Hanan 1990; Prusinkiewicz and Lindenmayer 1990; Hanan 1992) and makes it possible to specify simple models quickly and concisely. The evolution of the cpfg

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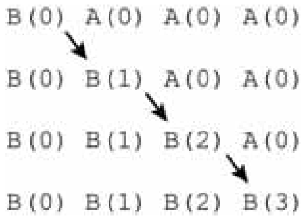
language has been surveyed by Prusinkiewicz (1999), with subsequent additions (in particular, decomposition and interpretation rules) described by Prusinkiewicz et al. (2000). The currently available constructs have been listed by Měch et al. (2005). The language of the lpfg simulator was designed more recently (Karwowski 2002; Karwowski and Prusinkiewicz 2003; Karwowski and Lane 2006) to facilitate the specification of complex plant models (e.g. Mündermann et al. 2005; Allen et al. 2005). We call this language L+C, because it combines features of L-systems and the C++ programming language. Here we overview key features of L+C in a manner that complements and updates its earlier presentation by Karwowski and Prusinkiewicz (2003), and we illustrate the discussed features in the context of a complete L+C program: a model of a growing branch that bends due to gravity.

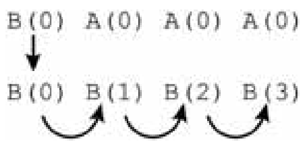
FAST INFORMATION TRANSFER

An essential component of functional-structural plant models is transport of information through the organism (Perttunen et al. 1996; Allen et al. 2005). A simple L-system example would be assigning consecutive numbers to a sequence of modules in a string. This task can be accomplished using the context-sensitive production `B(m) < A(n) → B(m+1)`, as illustrated below for a string of N=4 modules:



We observe that it takes N–1 derivation steps for the information to propagate to the end of an N-element string, as indicated by the arrows. This is a consequence of determining the next state of each module as a function of the current state of its left neighbour. Although each module in the string is rewritten in each derivation step, the state of only one module is changed. The remaining modules are not affected.

L+C includes a construct that makes it possible to accelerate this information transfer if the string is rewritten sequentially, from left to right, during a single derivation step. Under this assumption, the next state of a module can be calculated as a function of the (just calculated) next state of its left neighbour, rather than the current state of that neighbour. The flow of information is then represented by the arrows in the following derivation step:



In order to refer to this *new context* in L-system productions, we use the symbol << . A production that defines the above derivation can thus be written as

```
B(m) << A(n) → B(m+1).
```

In general, the predecessor of a production using new context in a derivation proceeding from left to right has the following format:

new-left-context << *strict-predecessor* > *right-context* .

In an analogous manner, if a derivation step is performed from right to left, a production predecessor with new context has the general format:

left-context < *strict-predecessor* >> *new-right-context* .

The *left-context* and *right-context* fields are optional. Motivated by the above example, we refer to L-systems that use the new context construct as L-systems with fast information transfer. Note that a production may only have one new context field (i.e., either << or >>), depending on the derivation direction. Productions with a new context field inconsistent with the current derivation direction are ignored.

FORMAT OF L+C PRODUCTIONS

The productions considered in the previous section were specified using a mathematical L-system notation. In L+C, the arrow is replaced by the keyword `produce`, which leads to production specification of the form

```
B(m) << A(n) : {produce B(m+1);}
```

In general, an L+C production has the syntax:

predecessor : {*production body*}

where the predecessor is specified as discussed in the previous section, and the production body is a block of C++ code including one or more statements:

`produce` *parametic-string*$_{opt}$;

Although the mathematically inspired arrow notation may appear more elegant than the L+C notation, the latter is more flexible as a programming construct. For example, an L+C production may include several alternative successors, as in the construct:

```
B(m) << A(n) : {
  m++;
  if (m%3==0) produce C(m)B(m); else produce B(m);
}
```

In addition to assigning consecutive numbers to consecutive modules `B(m)`, this production inserts a module `C(m)` in front of every `B(m)` each time `m` is divisible by 3. The traditional notation would require two separate productions to express the same idea, which might be inefficient if the computations preceding production application were much longer than the simple incrementation, `m++`.

L+C also supports a variant of the `produce` statement, denoted by keyword `nproduce`. In contrast to `produce`, the execution of `nproduce` does not terminate the application of a production. This makes it possible to construct the successor of a production 'piece by piece'. For example, using `nproduce`, the previous production can be simplified to the form:

```
B(m) << A(n) : {
  m++;
  if (m%3==0) nproduce C(m);
  produce B(m);
}
```

STRUCTURED PARAMETERS AND MODULE DECLARATIONS

In the above examples, we have only considered modules with a single numerical parameter. L+C extends the previous definition of parametric L-systems (Lindenmayer 1974; Prusinkiewicz and Hanan 1990; Prusinkiewicz and Lindenmayer 1990; Hanan 1992) by allowing for the use of parameters of different types, including user-definable data structures. To make this possible, L+C modules are declared before use. Declaration specifies the number and types of parameters that are associated with the given module type with the following syntax:

`module` *identifier* (*parameter-list*$_{opt}$) ;

To illustrate the usefulness of structured parameters, let us consider the following context-sensitive production in cpfg notation:

```
L(xl1,xl2,xl3,xl4,xl5) < A(x1,x2,x3,x4,x5) >
R(xr1,xr2,xr3,xr4,xr5) → A(x1,x2,x3,x4,x5+1)
```

This production operates on a module A with five real-valued parameters, and increments the value of the last parameter by 1 if module A appears in the context of modules L and R. Note that, due to the relatively high number of module parameters, this production is difficult to read. The corresponding L+C code is:

```
struct data { float x1, x2, x3, x4, x5; };
module A(data); module L(data); module R(data);
L(dl) < A(d) > R(dr) :
{
  d.x5 += 1.0;
  produce A(d);
}
```

In this example, as in most simple programs, the L+C code is longer than the equivalent cpfg code. Nevertheless, the use of structured module parameters offers several advantages:

- Long lists of parameter modules can be avoided. This results in a more legible code.
- The L+C code is less error-prone. In cpfg it is easy to introduce an error by inadvertedly skipping a parameter in a long parameter list.
- The L+C code is easier to modify. For example, if an additional parameter `x6` is needed to characterize modules `L`, `R` and `A`, in L+C it suffices to extend the definition of the structure `data`. In contrast, in cpfg it is necessary to include `x6` explicitly in the parameter list associated with each occurrence of these modules.

CONTROL OF L-SYSTEM PROGRAM EXECUTION

In principle, the notion of L-systems leads to a declarative programming style. Each production is a statement of the form: “If a module and its neighbours match the production predecessor then subsequent actions will be performed as specified in the production body”. These actions thus depend on the state and context of the module to which they apply, rather than a control flow mechanism as found in imperative languages.

Nevertheless, L+C also includes elements of the imperative programming style. The body of each production is specified as a sequence of statements based on C++. Furthermore, there are four blocks of statements that are performed at specific points of the derivation: at the beginning of the simulation (`Start`), at the beginning of each derivation step (`StartEach`), at the end of each step (`EndEach`), and at the end of the simulation (`End`). These blocks were already defined for cpfg (Hanan 1992), but play a more significant role in L+C because they may include statements that affect the flow of the simulation.

One such pair of statements are calls to the predefined functions `Forward()` and `Backward()`. These functions are typically used within the `StartEach` block, and determine whether the derivation will proceed left-to-right or right-to-left in the forthcoming step. As we have seen in the section "*Fast information transfer*", the direction of the derivation is of critical importance in the case of fast information transfer, because left new context can only be used when the derivation proceeds left-to-right, and right new context can only be used when the derivation proceeds right-to-left.

As the applicability of productions with new context depends on the direction of the derivation, only a subset of the production set may apply in a given derivation step. In the case of fast information transfer this subset may be established implicitly, by ignoring productions with the new context that are incompatible with the current direction of derivation. However, in many models it is convenient to control the applicable production set explicitly (Frijters and Lindenmayer 1974; Frijters 1976). To this end, L+C supports statements of the form

`group` *group_id* :

which divide the set of productions into subsets called groups (or tables in L-system theory; cf. Rozenberg 1973; Ginsburg and Rozenberg 1975). A `group` statement can be inserted before any production, and assigns a numerical label *group_id* to the subset of productions that follow. This label remains in effect until the next `group` statement or the end of the production list. The `group` statements are used in conjunction with the function

`UseGroup(`*group_id*`)`

which is typically called within the `StartEach` block and defines the group of productions to be used in the forthcoming step. By definition, productions in group 0 are used in all steps.

The notion of groups lends itself to the division of the simulation into a sequence of phases, each characterized by the use of a specific production group. The sequence of phases that constitute a simulation may be fixed, but it may also be determined dynamically, with the next phase depending on the outcome of the previous phase. For example, such a situation may occur if L-system productions are applied iteratively, until some criterion of convergence is met. It may then be desirable to interpret and visualize the result of a simulation graphically only after convergence has been achieved. To this end, L+C supports the function

```
DisplayFrame()
```

which is typically called within the `EndEach` block to display the result of the latest simulation step. Furthermore, as the number of iterations may be difficult to define *a priori*, L+C supports the function

```
Stop()
```

which terminates the execution of the simulation at the end of the derivation step in which it has been called. Examples of the constructs discussed in this and the following sections will be given in the context of a complete L+C model (Section "*A Biomechanical example*").

MULTIVIEW VISUALIZATION

In some applications, it is useful to display different aspects (views) of a simulation concurrently (Roberts 2000). For example, one view may realistically represent a developing plant, while another shows corresponding statistical information in the form of a dynamically updated table or a histogram. In L+C, different views can be displayed in separate windows, the contents of which are specified using subsets of interpretation rules (Prusinkiewicz et al. 2000). Each subset is called a visual group, and is identified by the statement

`vgroup` *view* :

A `vgroup` statement assigns the label *view* to the subset of interpretation rules that follow it. A visual group is terminated by the next `vgroup` statement or the end of the production list.

An important difference between the `group` and `vgroup` statements concerns the execution of the affected productions. In addition to group 0, only one production group, specified by the latest call to the `UseGroup()` function, applies to any particular simulation step. In contrast, interpretation rules in several visual groups can be executed in each simulation step, provided that windows associated with these groups are open. An L+C programmer may control which windows are open using the function call

`UseView(`*view*`).`

Additional constructs are provided in L-studio and lpfg to determine the default size and position of the windows, and to open and close them using menus.

INTERACTION WITH THE MODELS

Computational models are often used in simulated experiments in which model attributes are modified to address ‘what if’-type questions. Models expressed in L+C are conducive to such experiments, since the user can modify any aspect of the model by changing the L-system code. In addition, L-studio and vlab provide interactive tools that provide an interface for manipulating the numerical parameters and functions incorporated in the model. These tools include user-configurable virtual control panels and graphical function editors (Prusinkiewicz 2004).

The user may also explore models by directly manipulating their visual representations on the screen. Such manipulations may mimic physical operations such as pruning, girdling or pulling branches. The fundamental operation underlying these manipulations is the selection of a module within the graphical representation of the model. When the user selects a module with the mouse, a reserved module, `MouseIns()` in L+C, is automatically inserted before the symbolic representation of the selected module in the L-system string. The modeller specifies the response to this event using a production that includes `MouseIns()` in the predecessor.

A BIOMECHANICAL EXAMPLE

To illustrate L+C constructs in the context of a complete program, let us consider the biomechanical model of a growing pendulous branch proposed by Jirasek et al. (2000) according to the ideas of Fournier (1989). Jirasek et al. observed that the forces and torques involved in the bending, as well as the resulting reorientations and displacements of internodes, can be considered signals that propagate between plant modules and have local effects. This observation led to L-system implementations of the biomechanical model, first in cpfg (Jirasek 2000) and later in L+C (Taylor-Hell 2005). The L+C implementation makes use of almost all the programming constructs specific to this language, and therefore provides a good example of its features. In the model below, we ignore for simplicity the effects of tropisms and secondary growth. We also assume that the model operates in 2D.

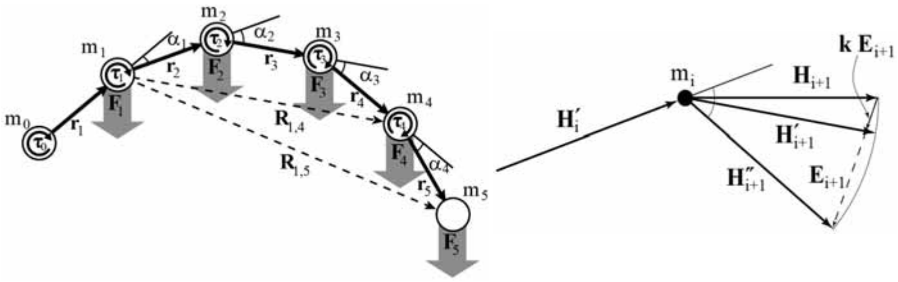


Figure 1. *Geometry of branch bending. Left: representation of a branch with N=5 internodes, and the symbols used in the derivation of the formula for static equilibrium. Right: the adjustment of internode orientation during the relaxation process*

The point of departure in the model construction is the derivation of equations that characterize branch shape in static equilibrium. We represent the branch as a sequence of internodes (Figure 1a), beginning with a fixed internode and terminated by an apex. The internodes are numbered from 1 (fixed proximal internode) to N (distal internode), and connect nodes numbered 0 to N. Each internode is represented as a vector $\mathbf{r}_i = s_i\mathbf{H}_i$, where the number s_i denotes internode length, and the unit vector $\mathbf{H}_i$ denotes internode orientation. The position of node j with respect to node i $(0 \le i < j \le N)$ is thus described by the vector

$$\mathbf{R}_{i,j} = \sum_{k=i+1}^{j} \mathbf{r}_k \ . \tag{1}$$

We assume that the branch would be straight in the absence of gravity but bends downward in the presence of gravity. To model this bending, we assume that each node i is an elastic joint, subject to a torque $\boldsymbol{\tau}_i$. This torque is caused by gravity acting on the overhanging part of the branch (positioned distally with respect to node i). To simplify calculations, we assume that the mass of the branch is concentrated in its nodes.

Let $\mathbf{F}_i = m_i\mathbf{g}$ denote the force acting on node i with mass m_i under gravitational acceleration $\mathbf{g}$. The torque $\boldsymbol{\tau}_i$ is the sum of the torques exerted by the individual masses positioned distally with respect to node i:

$$\boldsymbol{\tau}_i = \sum_{j=i+1}^{N} \mathbf{R}_{i,j} \times \mathbf{F}_j \ . \tag{2}$$

The above equation expresses torques in global terms, in the sense that it incorporates influences of masses m_i that may be positioned far away from node i. Nevertheless, torques acting on consecutive joints can be related to each other in local terms. Specifically, the torque acting on node $i-1$ is equal to

$$\boldsymbol{\tau}_{i-1} = \sum_{j=i}^{N} \mathbf{R}_{i-1,j} \times \mathbf{F}_j = \mathbf{R}_{i-1,i} \times \mathbf{F}_i + \sum_{j=i+1}^{N} \mathbf{R}_{i-1,j} \times \mathbf{F}_j = \mathbf{r}_i \times \mathbf{F}_i + \sum_{j=i+1}^{N} \left(\mathbf{r}_i + \mathbf{R}_{i,j}\right) \times \mathbf{F}_j$$
$$= \mathbf{r}_i \times \sum_{j=i}^{N} \mathbf{F}_j + \sum_{j=i+1}^{N} \mathbf{R}_{i,j} \times \mathbf{F}_j = \mathbf{r}_i \times \mathbf{T}_i + \boldsymbol{\tau}_i , \quad (3)$$

where

$$\mathbf{T}_{i-1} = \sum_{j=i-1}^{N} \mathbf{F}_j = \mathbf{F}_{i-1} + \sum_{j=i}^{N} \mathbf{F}_j = \mathbf{F}_{i-1} + \mathbf{T}_i \ . \quad (4)$$

In static equilibrium, a torque of magnitude $\tau_i = |\boldsymbol{\tau}_i|$, acting on the node *i*, rotates internode $\mathbf{r}_{i+1}$ by angle $\alpha_i = \tau_i / \kappa_i$ with respect to internode $\mathbf{r}_i$. The parameter κ_i is the rotational spring constant associated with node *i*. The geometry of the branch is thus described by the equations:

$$\begin{aligned} \mathbf{H}_{i+1} &= \text{Rotate}(\mathbf{H}_i, \alpha_i), \\ \mathbf{P}_{i+1} &= \mathbf{P}_i + s_i \mathbf{H}_i, \end{aligned} \quad (5)$$

where the function $\text{Rotate}(\mathbf{H}_i, \alpha_i)$ changes the orientation of vector $\mathbf{H}_i$ by angle α_i (in 2D), and $\mathbf{P}_i$ is the position of mass m_i.

To find the equilibrium, we use a relaxation method (Press et al. 1992, pp. 754-755), which in this case consists of an iterative application of two steps:

Step 1. Given a sequence of internodes $\mathbf{r}_i = s_i \mathbf{H}_i$, calculate torque τ_i acting on each node. To this end, scan the branch in the proximal direction, and apply Equations 3 and 4 to consecutive nodes.

Step 2. Given the torques τ_i, adjust the orientation of each internode. To this end, scan the branch in the distal direction. Given the adjusted orientation $\mathbf{H}_i^{'}$ of internode *i*, first find the vector $\mathbf{H}_{i+1}^{''}$ that forms angle $\alpha_i = \tau_i / \kappa_i$ with respect to $\mathbf{H}_i^{'}$ (Figure 1b). The vector $\mathbf{E}_{i+1} = \mathbf{H}_{i+1}^{''} - \mathbf{H}_{i+1}$ is the difference between the orientation of internode $i+1$ calculated in the previous iteration step and the orientation that would be required to achieve equilibrium. The adjusted orientation of internode $i+1$ is then defined as $\mathbf{H}_{i+1}^{'} = \text{Normalize}(\mathbf{H}_{i+1}^{'} + k\mathbf{E}_{i+1})$. Parameter *k* controls the amount of adjustment and thus affects the speed of convergence to the solution, and function Normalize restores the result to the unit length. Furthermore, the magnitudes of difference vectors are accumulated to form an error measure:

$$e = \sum_{i=0}^{N} |\mathbf{E}_i| \ . \quad (6)$$

Figure 2. *The sequence of phases in the simulation of a growing and bending branch. Similar graphs are useful when defining the sequence of phases in other L+C models as well*

The iteration ends when the error *e* falls below an assumed tolerance threshold. Within this tolerance, the orientations of the internodes and positions of the nodes then satisfy Equations 5. In the complete model of a growing branch, this is an appropriate time to simulate a developmental step. The cycle of computation is illustrated in Figure 2. The resulting L+C program is listed on the following pages.

The program begins with the definitions of constants, variables and functions, and declarations of data structures and modules (lines 1 to 53). The three-dimensional vector `V3f`, which appears for the first time in the definition of the gravity vector (line 10), is one of the types declared in the L+C header file `lpfgall.h`. An example of a user-defined function follows (lines 12-22). Lines 55-97 present a typical example of the organization of computation. The statements in the `Start` block (lines 57-67) initialize variables, set the initial phase of the computation, and activate two different views of the model at the beginning of the simulation. The statements in the `StartEach` block (lines 69-77) specify the production group and derivation direction to be used in the forthcoming simulation step. Finally, the statements in the `EndEach` block (lines 79-97) determine the sequence of simulation phases according to Figure 2. As the number of iterations needed to reach the equilibrium is not known in advance, the results are displayed on demand using the `DisplayFrame()` function (line 89), once the error drops below a threshold value `maxerror` (line 86). For similar reasons, the number of simulation steps needed for the branch to grow to a desired length is not known in advance. The simulation is thus terminated by a call to the `Stop()` function once the branch has reached the maximum prescribed length (lines 136-139). The `derivation length` statement (line 99) serves as a safeguard, specifying an upper limit on the number of steps.

The `Axiom` statement (line 100) specifies the initial structure. First is the `Axes()` module, which is used to draw coordinate axes in the `Torques` view. Following it is the branch, which initially consists of a single internode followed by an apex.

The L-system productions are divided into three groups. The `PropagateLeft` and `PropagateRight` groups (lines 102-127), with a single production each, iteratively compute the shape of the branch according to the mathematical analysis presented earlier in this section. The conciseness of these productions illustrates the expressive power of key constructs of L+C: context-sensitive productions, fast information transfer, modules with structured parameters, and vector operations.

```
#include <math.h>       // C/C++ math header
#include <lpfgall.h>    // declarations: L+C variables,
                        // structures, functions, etc.
const int MAX_N=30;     // max number of internodes
const float S=1.0;      // internode length
const float MASS=0.1;         // mass of a node
const float KAPPA=4500.0;     // spring constant
const float maxerror = 0.01;  // error limit
const float relax=0.5;  // relaxation coefficient
const V3f Gravity(0,-9.81,0); // gravity vector

/* Sample definition of functions on vectors:
   Rotation of vector a in xy plane by angle alpha */

V3f VecRotate(V3f a, float alpha)
{
  V3f c;
  c.x = a.x * cos(alpha) - a.y * sin(alpha);
  c.y = a.x * sin(alpha) + a.y * cos(alpha);
  c.z = 0;
  return c;
}

/* Analogous definitions of cross product, vector
   length, and vector normalization should go here */

/* Declarations of structures, modules, and variables */

struct InternodeData
{
  float s;           // internode length
  float mass;        // node mass
  float kappa;// rotational spring constant
  float sigma_mass; // total mass to the right
  float torque;      // torque from masses to the right
  V3f P;             // proximal node position
  V3f H;             // internode orientation
};

module Internode(InternodeData);
module Apex(int);    // int = number of internodes
module Axes();// for visualizing coordinate axes

InternodeData iid;  // initial internode data
int Phase;          // computation phase
float error;        // distance from equilibrium
int color;          // current color index

/* Enumeration of computation phases */

#define PropagateLeft   1 // accumulate masses, torques
#define PropagateRight  2 // update angles, positions
#define Grow            3 // append internode

/* Organization of computation */

Start:           // At the beginning of simulation
{                // initialize non-zero variables:
  iid.s = S;               // internode length,
  iid.mass = MASS;         // node mass,
  iid.kappa = KAPPA;       // rot. spring constant,
  iid.H.x = 1;             // internode orientation,
```

```
  Phase = PropagateLeft;   // initial phase,
  color = 1;               // initial color index.
  UseView(Branch);         // Display view "Branch"
  UseView(Torques);        // Display view "Torques"
}

StartEach:       // At the beginning of simulation step
{                // set group and derivation direction:
  UseGroup(Phase);            // set current group,
  if (Phase == PropagateLeft) // depending on the phase
    Backward();               // derive right-to-left
  else                        // or
    Forward();                // left-to-right.
  error = 0;     // Also, clear cumulative error.
}

EndEach:         // At the end of simulation step
{                // determine next phase:
  switch (Phase) {            // consider prev. phase;
  case PropagateLeft:         // after propagating left
    Phase = PropagateRight;   // propagate right,
    break;
  case PropagateRight:        // after propagating right
    if (error > maxerror)     // if error above limit
      Phase = PropagateLeft;  // propagate left
    else {                    // otherwise
      DisplayFrame();         // display branch
      Phase = Grow; }         // and grow,
    break;
  case Grow:                  // after growing
    Phase = PropagateLeft;    // propagate left;
    ++color;                  // increment color index,
    break;
  }
}

derivation length: 1000000;
Axiom: Axes() Internode(iid) Apex(0);

/* Accumulate masses and torques using fast information
   transfer to the left. */

group PropagateLeft:
Internode(id) >> Internode(idr) :
{
  id.sigma_mass = id.mass + idr.sigma_mass;
  id.torque = idr.torque +
    VecCrossProd(idr.H, id.sigma_mass*Gravity).z ;
  produce Internode(id) ;
}

/* Update node angles and internode positions using
   fast information transfer to the right. */

group PropagateRight:
Internode(idl) << Internode(id) :
{
  V3f NewEquilVector = VecRotate(idl.H,
                                 id.torque/id.kappa);
  V3f DifferenceVector = NewEquilVector - id.H;
  error += VecLength(DifferenceVector);
  id.H = VecNormalize(id.H + relax*DifferenceVector);
```

```
125    id.P = idl.P + id.s*id.H;
126    produce Internode(id);
127  }
128
129  /* Append new internode unless maximum number reached.
130     Handle interaction with the model. */
131
132  group Grow:
133  Internode(id) < Apex(n):
134  {
135    id.P += id.s*id.H; // Calculate new internode position
136    if (n<=MAX_N)      // if internode limit not exceeded
137      produce Internode(id) Apex(n+1);  // append
138    else               // otherwise
139      Stop();          // terminate simulation.
140  }
141
142  MouseIns() Internode(id): // If node selected by mouse
143  {
144    id.mass *= 3 ;          // increase its mass 3 times.
145    produce Internode(id);
146  }
147
148  /* Visualize simulation results */
149
150  interpretation:
151  group 0:          // display irrespective of phase
152
153  vgroup Branch :   // specification of view "Branch":
154  Internode(id) :   // draw an internode as a line
155  {                 // and a circle
156    nproduce SetColor(color);
157    produce LineTo3f(id.P) Circle(id.mass);
158  }
159
160  vgroup Torques:   // specification of view "Torques":
161  Internode(id) :   // extend the plot of torques
162  {
163         id.P.y = -0.045*id.torque;
164         produce SetColor(color) LineTo3f(id.P);
165  }
166
167  Axes() :          // draw coordinate axes
168  {
169         nproduce SetColor(255) SB() F(20) EB();
170         produce SB() Right(90) F(25) EB();
171  }
```

The `Grow` group consists of two productions. The first production (lines 133-140), describes the addition of a new internode to the branch and stops the simulation once the maximum branch length has been reached. The second production (lines 142-146) handles interaction with the model. If the user clicks on an internode, the symbol `MouseIns()` is automatically inserted before the corresponding `Internode` module in the L-system string. The production handles this situation by incrementing the mass of the internode threefold and removing the `MouseIns()` symbol from the string. This operation can be used to investigate the impact of a fruit load on the branch shape, for example.

The remainder of the code (lines 148-171) is devoted to the visualization of simulation results. Two windows are used for this purpose. The first window is associated with the `vgroup Branch` and shows the shape of the growing branch. The second window is associated with the `vgroup Torques` and shows the distribution of torques along consecutive nodes of the branch. These views are obtained using different interpretations of the same module `Internode`. The last rule, with predecessor `Axes()` defined in the axiom, plays an auxiliary role of displaying coordinate axes in the plot of torques. With a few additional lines of code, we could also place numerical scales and labels on the axes.

A snapshot of simulation results is shown in Figure 3.

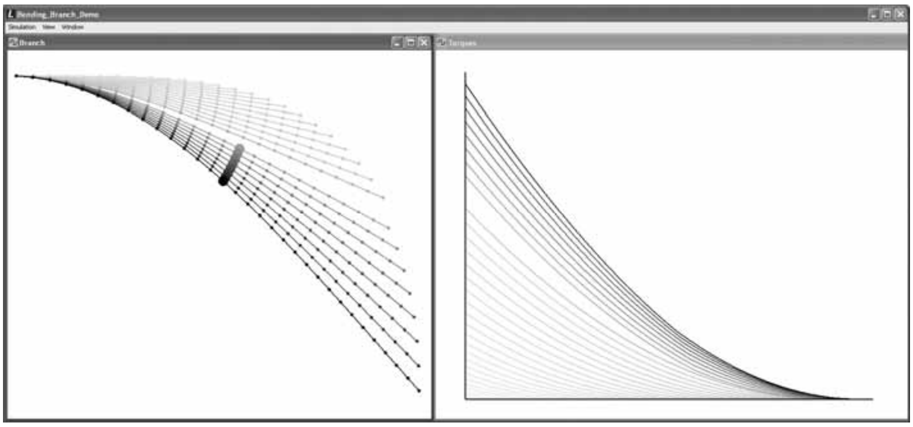


Figure 3. *Visualization of the model of a growing and bending branch. The simulation was run in a 'stroboscopic' mode, in which consecutive simulation stages are superimposed. Left view: changes of the branch shape over time. The mass of the node represented by the enlarged circle has been increased interactively. Right view: distribution of torques along the branch. Grey levels represent developmental stages (lengths) of the branch, and visually associate branch shapes with the corresponding torque plots*

CONCLUSIONS

The most significant conceptual advancement in L+C, compared to previous L-system-based languages, is fast information transfer. It significantly speeds up many simulations, especially those of functional-structural and biomechanical models, which rely on the propagation of hormones, resources or mechanical forces through the plant. In addition, the specification of complex models is facilitated through the use of modules with structured parameters, and the division of productions into groups. L+C also provides the modeller with the wealth of programming constructs available in C++. The biomechanical model of a growing pendulous branch presented in the section "*A Biomechanical example*" illustrates the use of these features in the context of a complete L+C program.

L+C is well suited to the specification of models that incorporate a single aspect of plant function within a developing plant structure. An open problem is the construction of comprehensive models that incorporate several aspects, such as genetics, partitioning of different resources, hormonal control, biomechanics and development. The challenge is to devise a methodology, supported by appropriate language constructs, that would make it possible to build such models in a well-structured manner – with different model components specified, implemented and tested independently, and easily combined into final synthetic models.

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