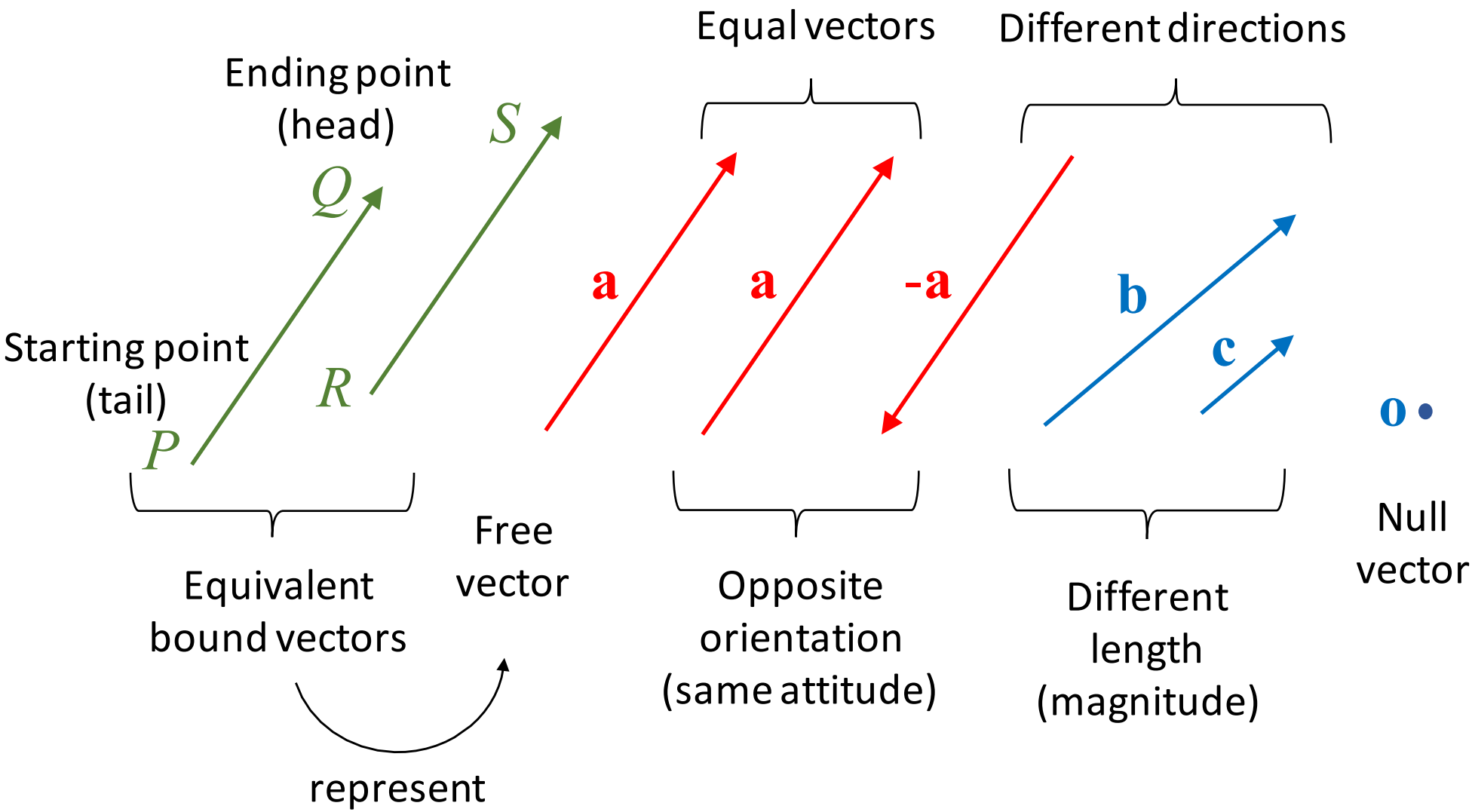
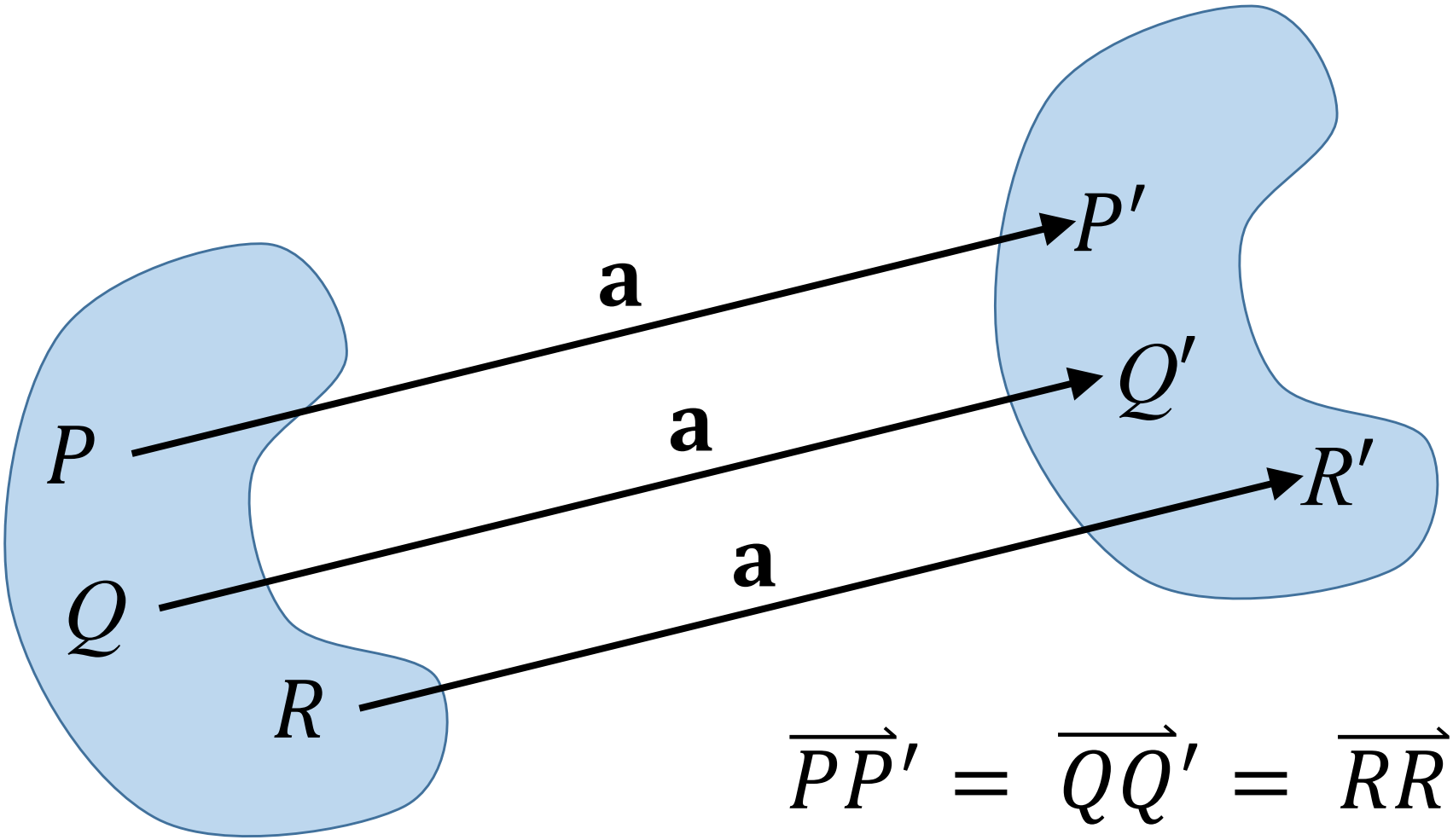
Examples of vectors and relations between them

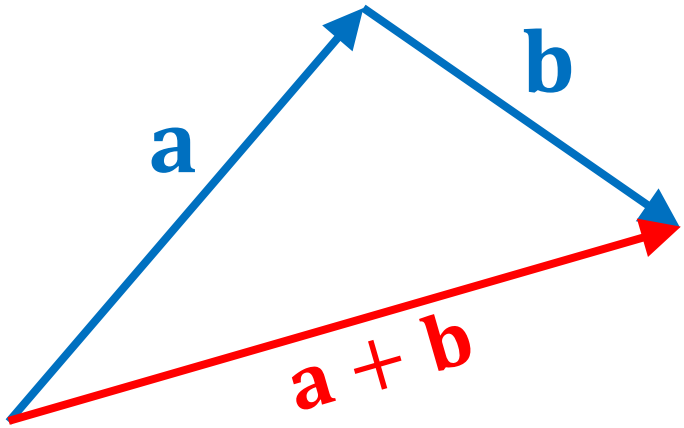


Interpretation of a vector as a translation of a rigid body

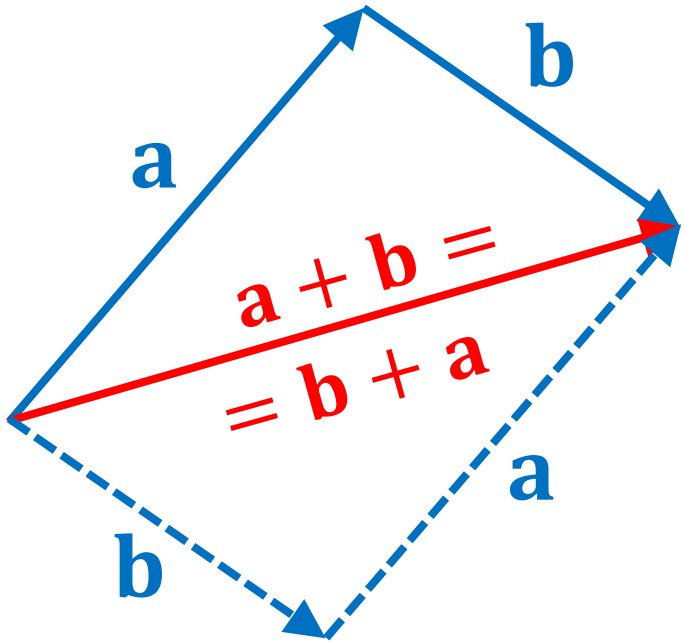


$$\overrightarrow{PP'} = \overrightarrow{QQ'} = \overrightarrow{RR'} = \mathbf{a}$$

Definition of vector addition

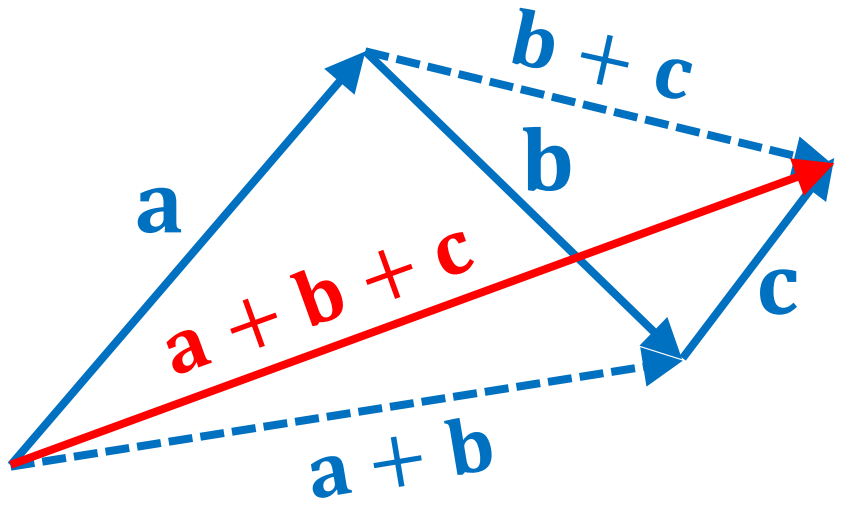


Commutativity of vector addition



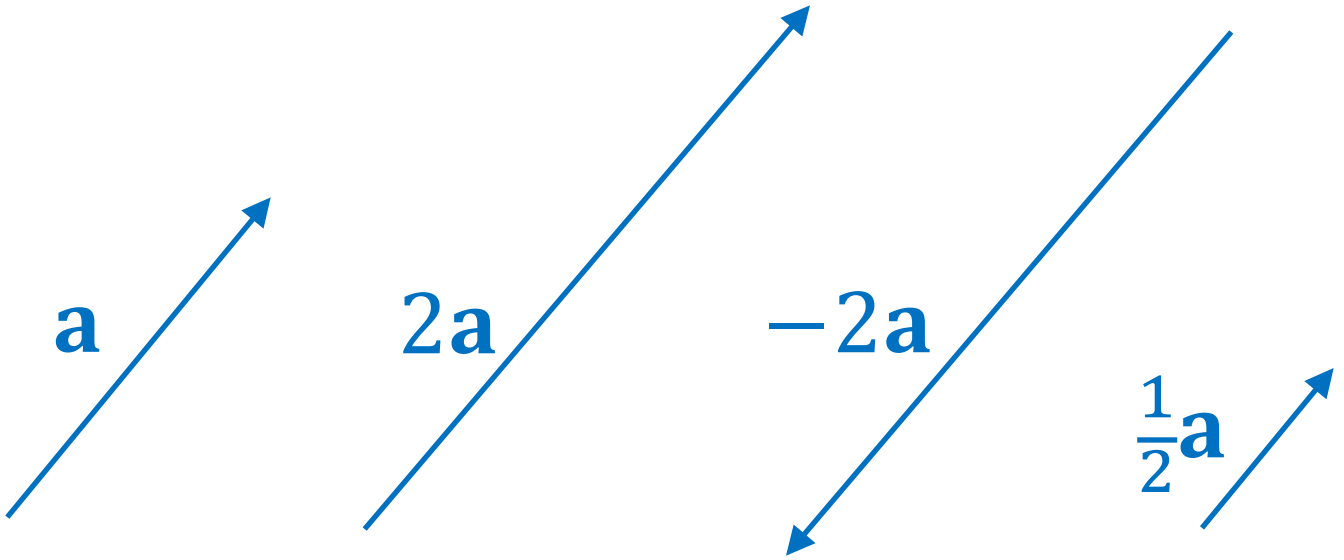
$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

Associativity of vector addition

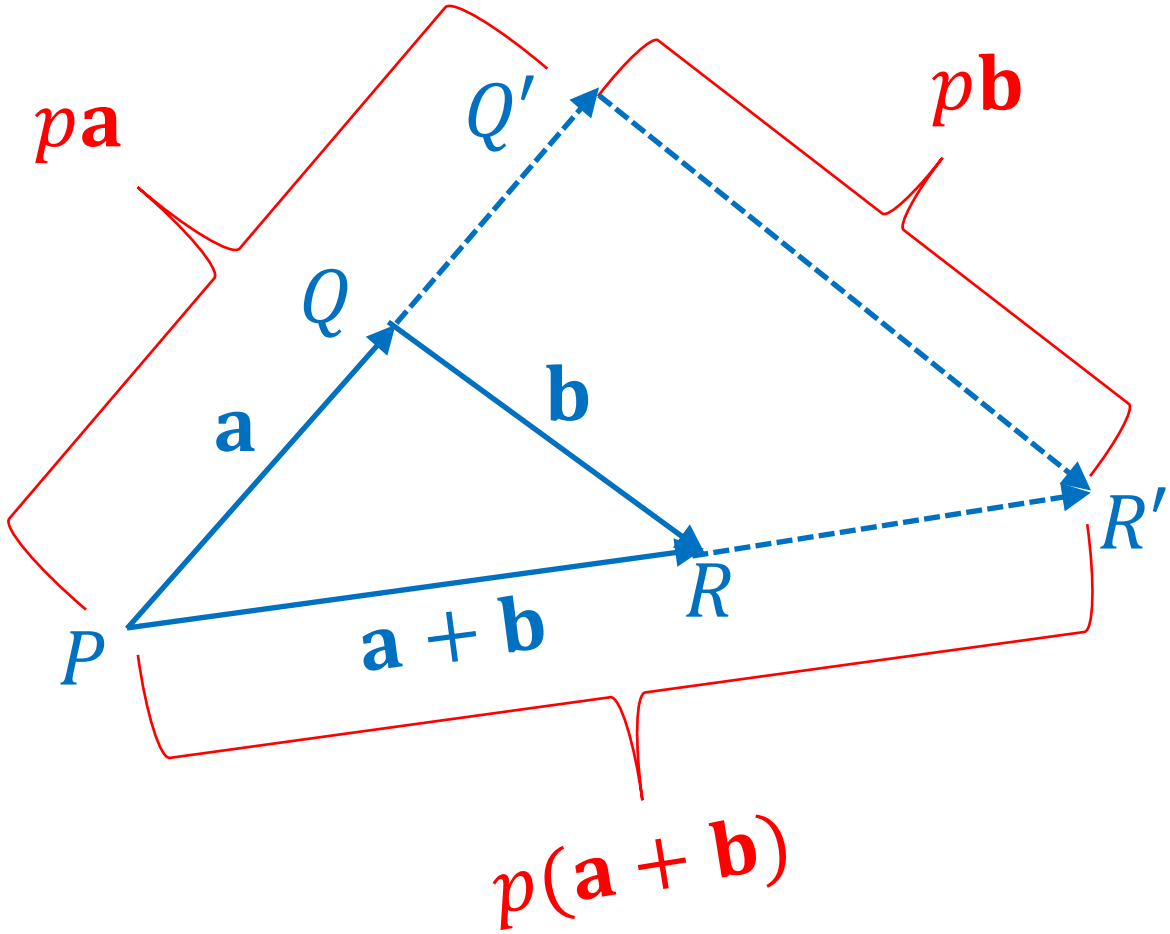


$$(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$$

Multiplication of a vector by a scalar



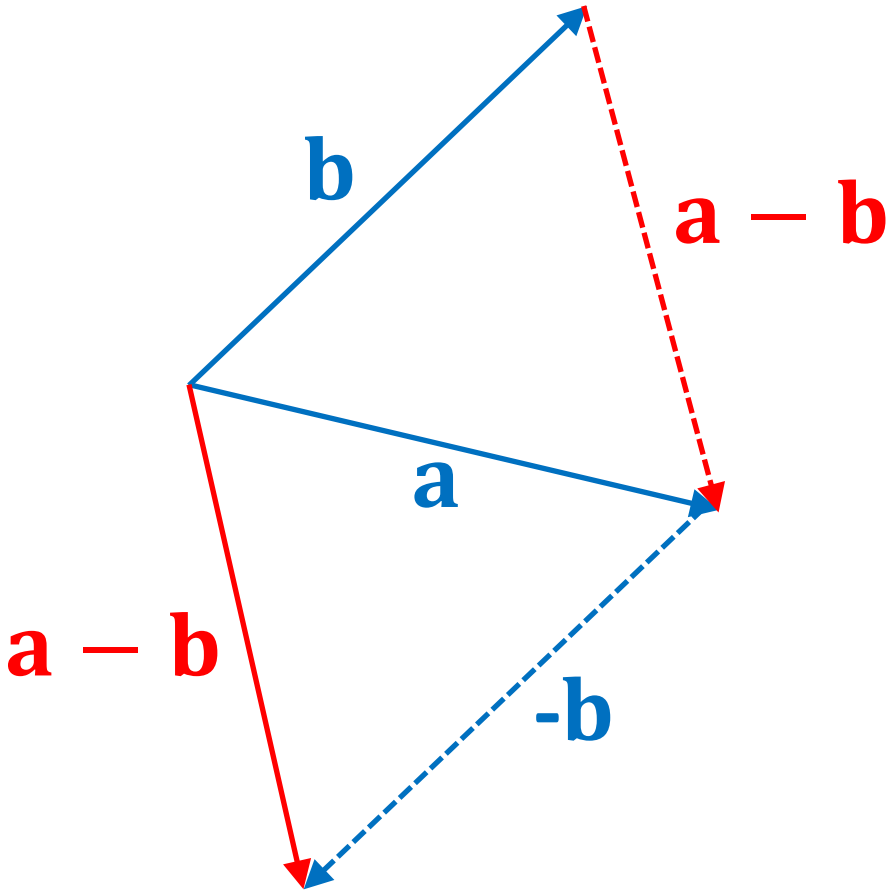
Distributive property



$$p(\mathbf{a}+\mathbf{b}) = p\mathbf{a} + p\mathbf{b}$$

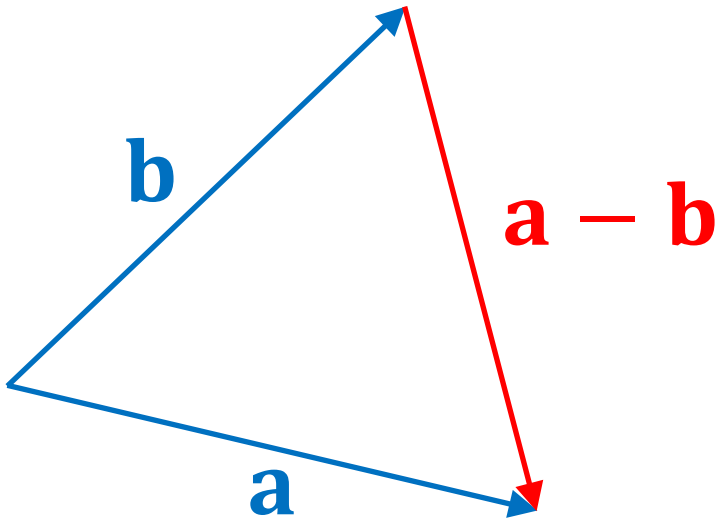
also $(p+q)\mathbf{a} = p\mathbf{a} + q\mathbf{a}$

Definition of vector difference

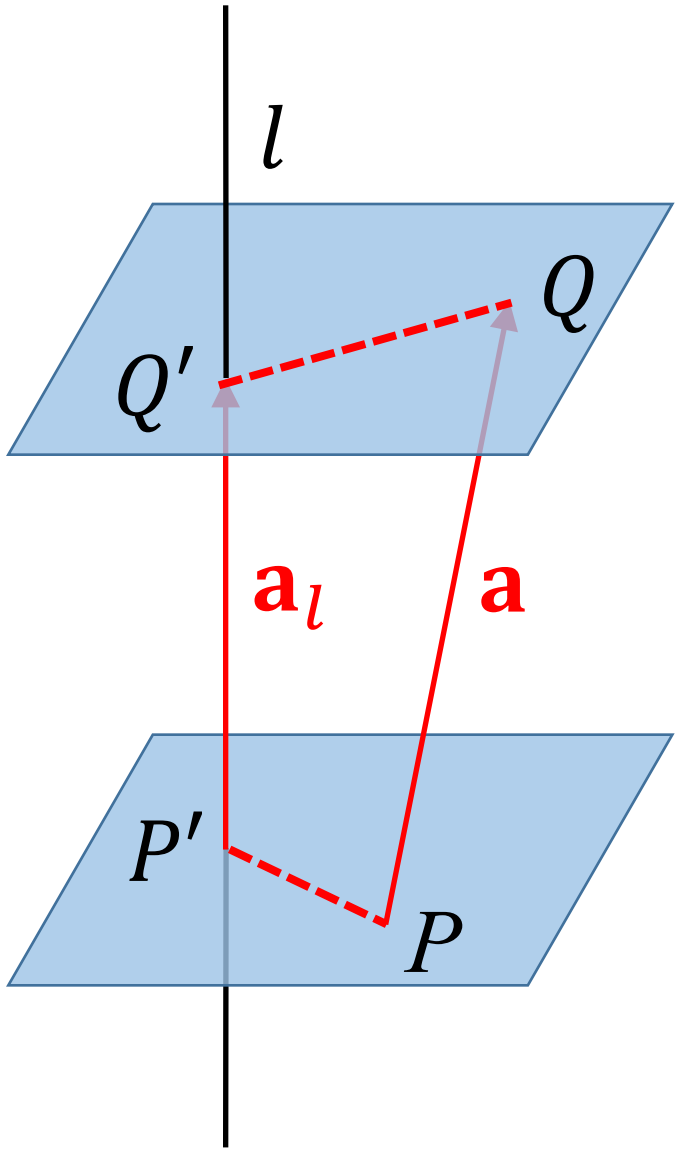


$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-1)\mathbf{b}$$

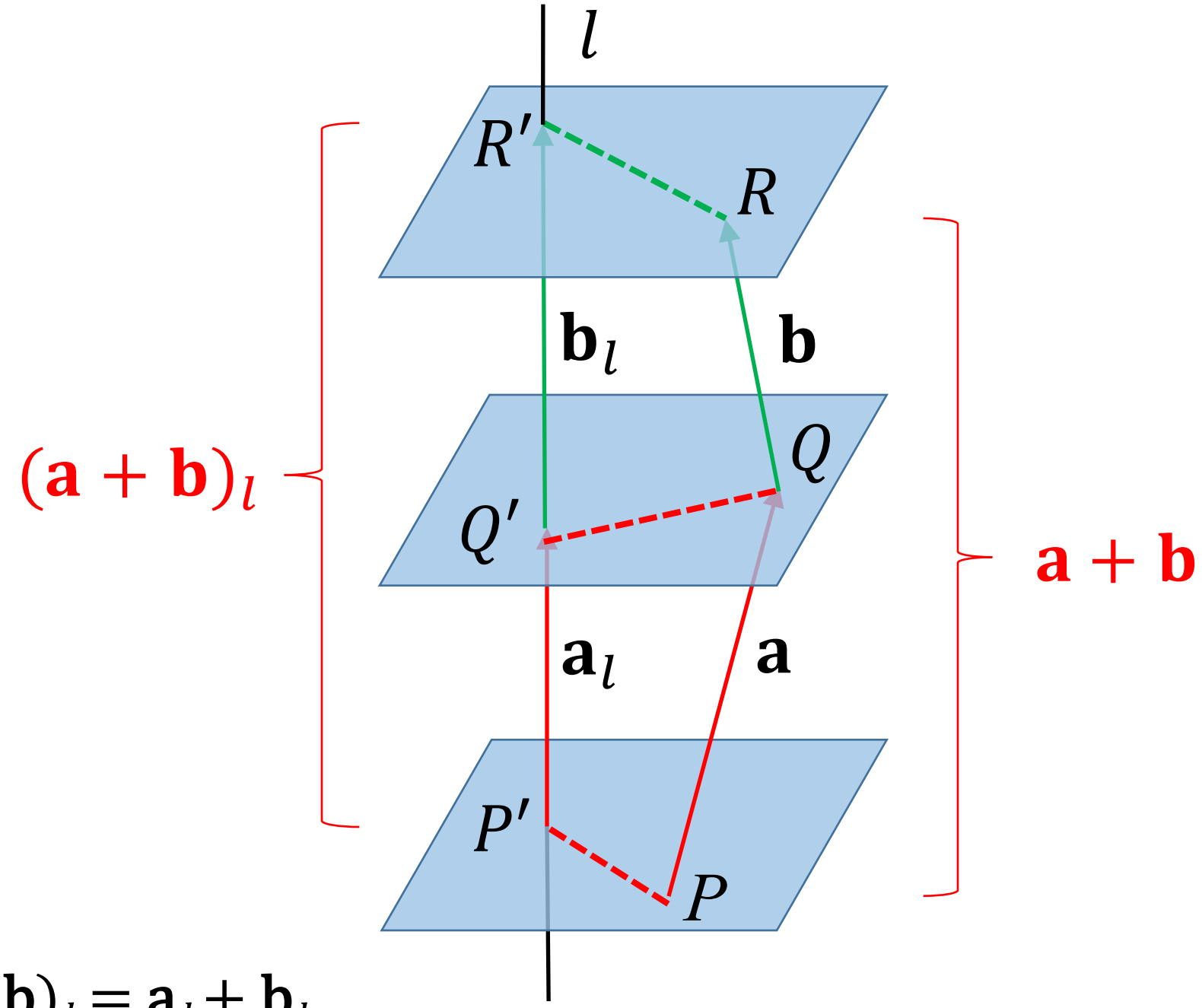
The usual construction of vector difference



Projection of a vector on a line

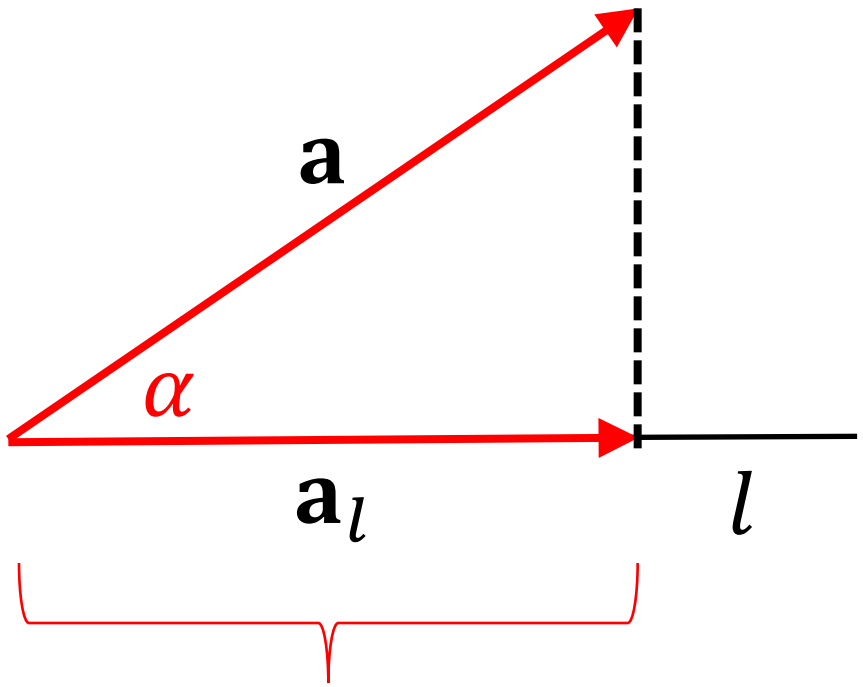


Distributive property



$$(\mathbf{a}+\mathbf{b})_l = \mathbf{a}_l + \mathbf{b}_l$$

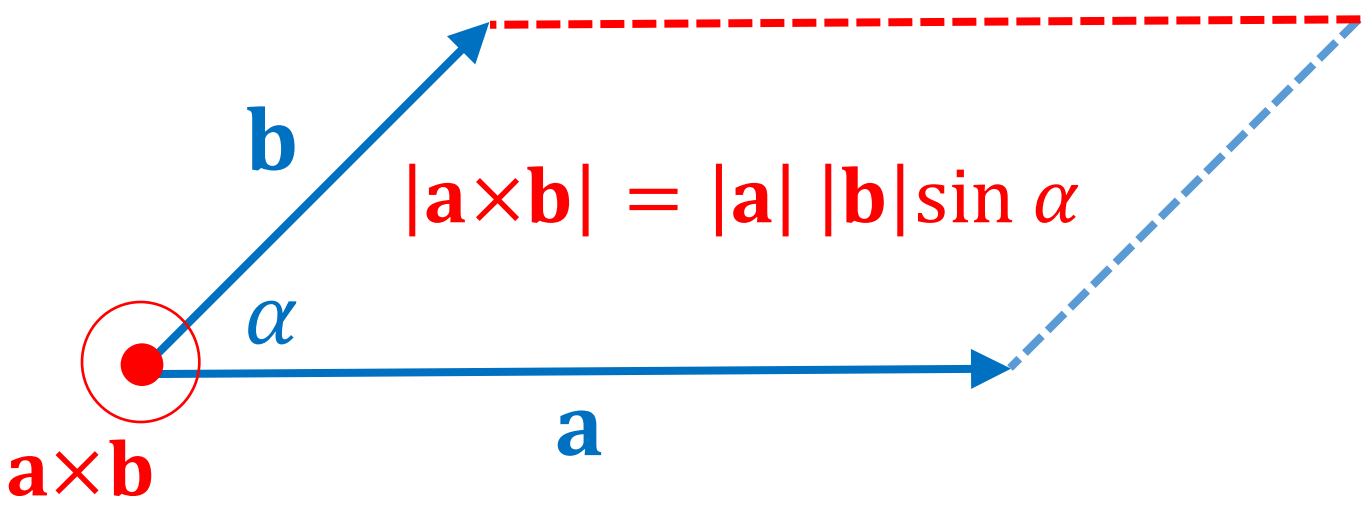
Calculating projection length



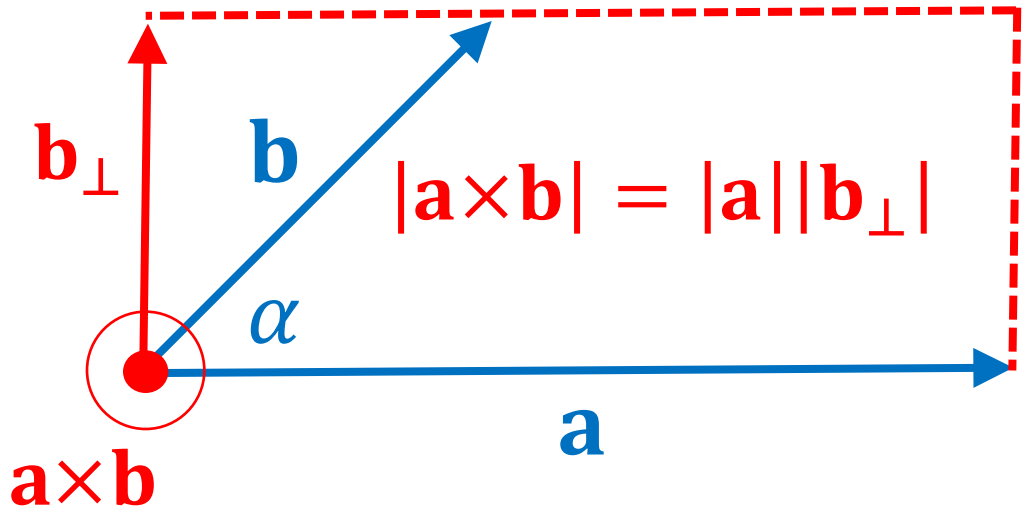
$$a_l = |\mathbf{a}_l| = |\mathbf{a}| \cos \alpha$$

Cross product

(a)

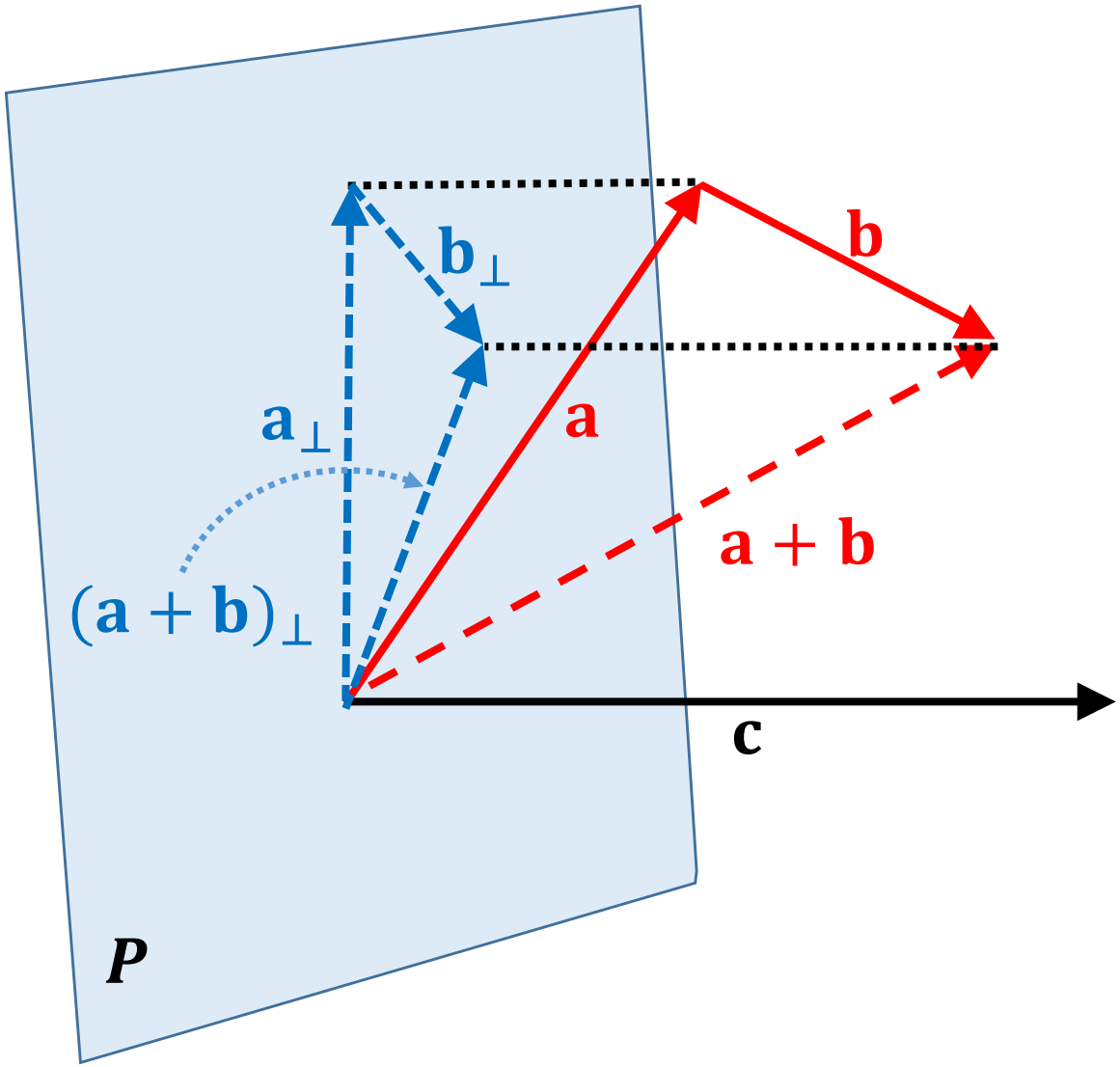


(b)



$$\mathbf{a}\times\mathbf{b} = \mathbf{a}\times\mathbf{b}_{\perp}$$

Toward a distributive property

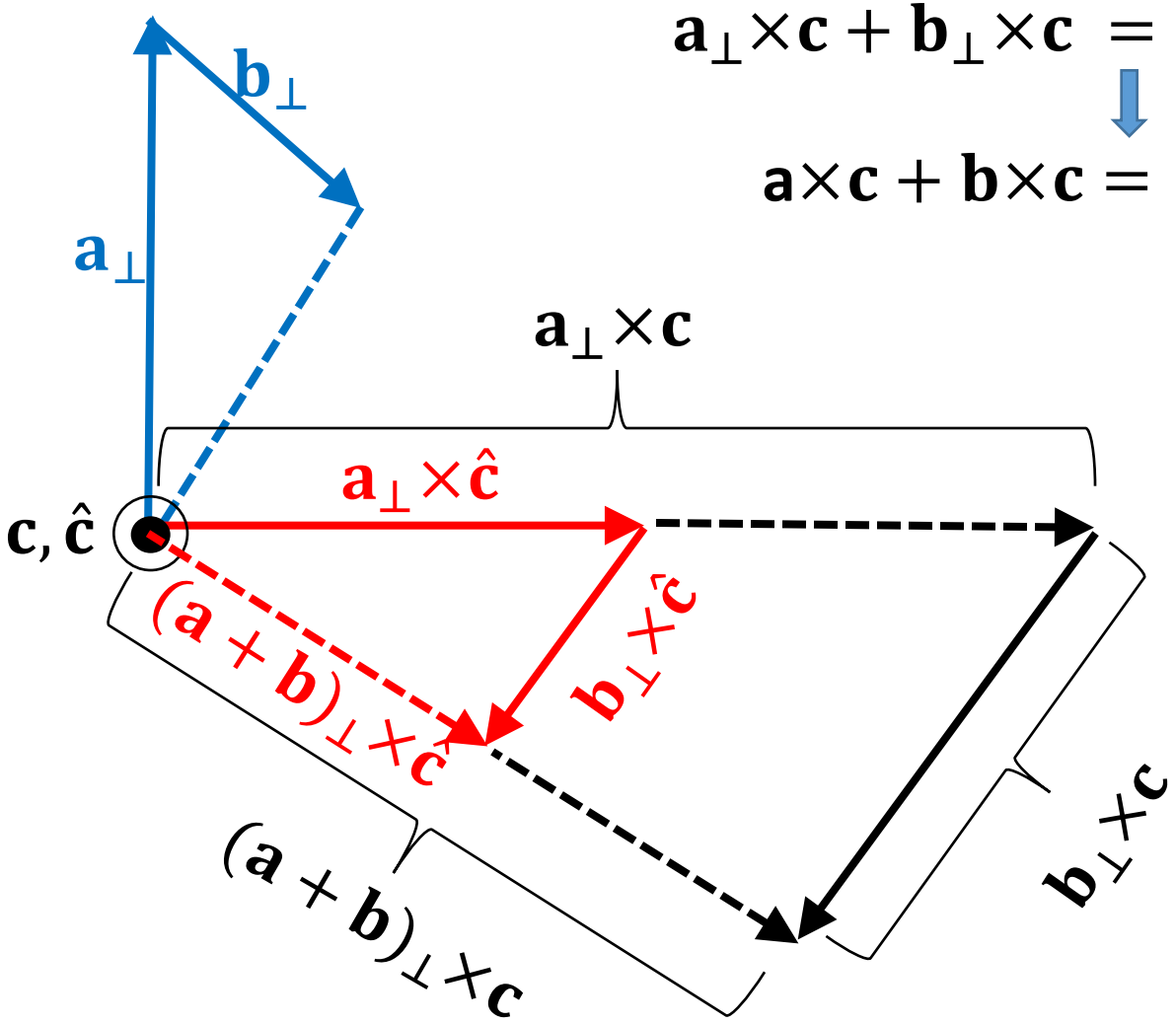


The distributive property

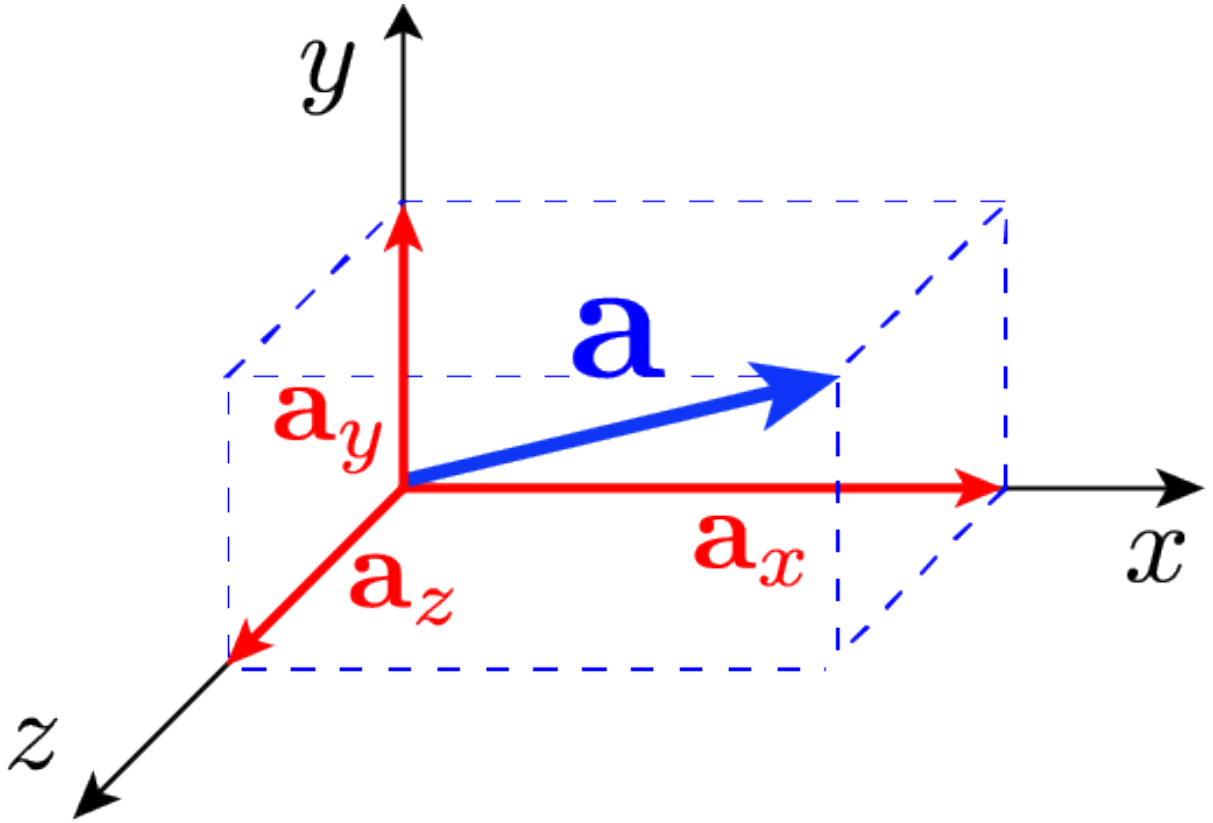
$$\mathbf{a}_\perp \times \hat{\mathbf{c}} + \mathbf{b}_\perp \times \hat{\mathbf{c}} = (\mathbf{a}+\mathbf{b})_\perp \times \hat{\mathbf{c}}$$

$$\mathbf{a}_\perp \times \mathbf{c} + \mathbf{b}_\perp \times \mathbf{c} = (\mathbf{a}+\mathbf{b})_\perp \times \mathbf{c}$$

$$\mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c} = (\mathbf{a}+\mathbf{b}) \times \mathbf{c}$$



Vector in coordinates



$$\mathbf{a} = \mathrm{a}_x\,\hat{\boldsymbol{\imath}} + \mathrm{a}_y\,\hat{\boldsymbol{\jmath}} + \mathrm{a}_z\,\hat{\boldsymbol{k}}$$